Cambridge International AS & A Level
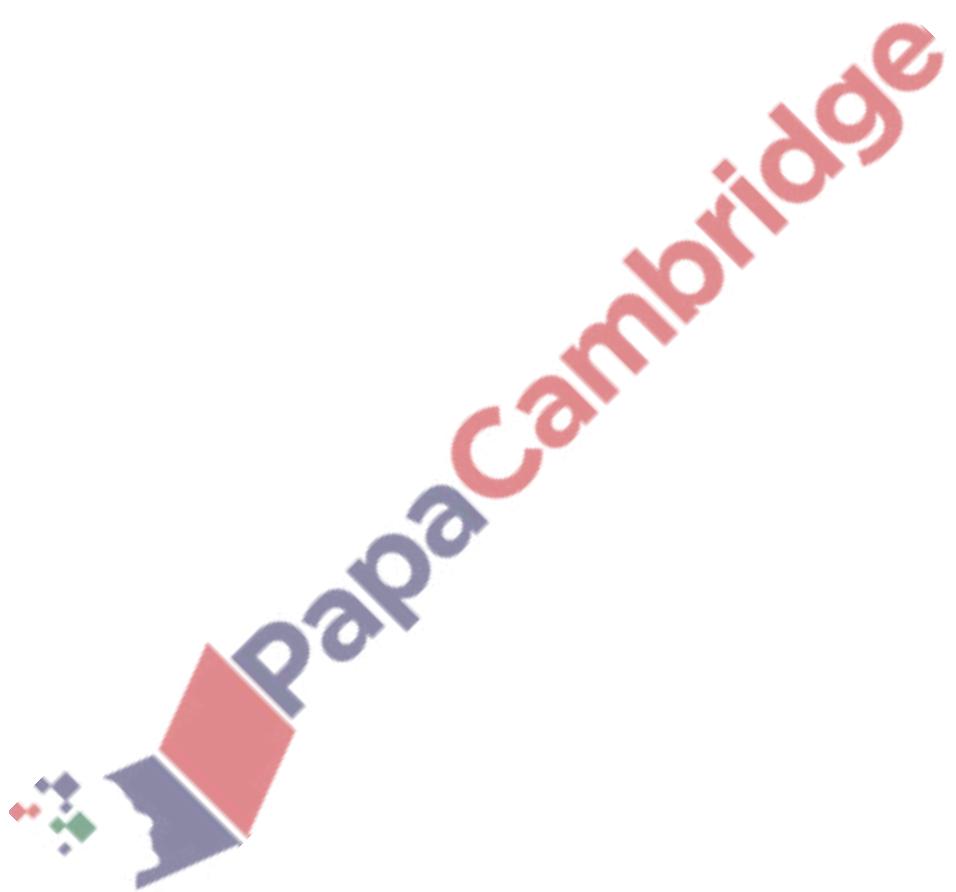
MATHEMATICS (9709) P2

TOPIC WISE QUESTIONS + ANSWERS | COMPLETE SYLLABUS
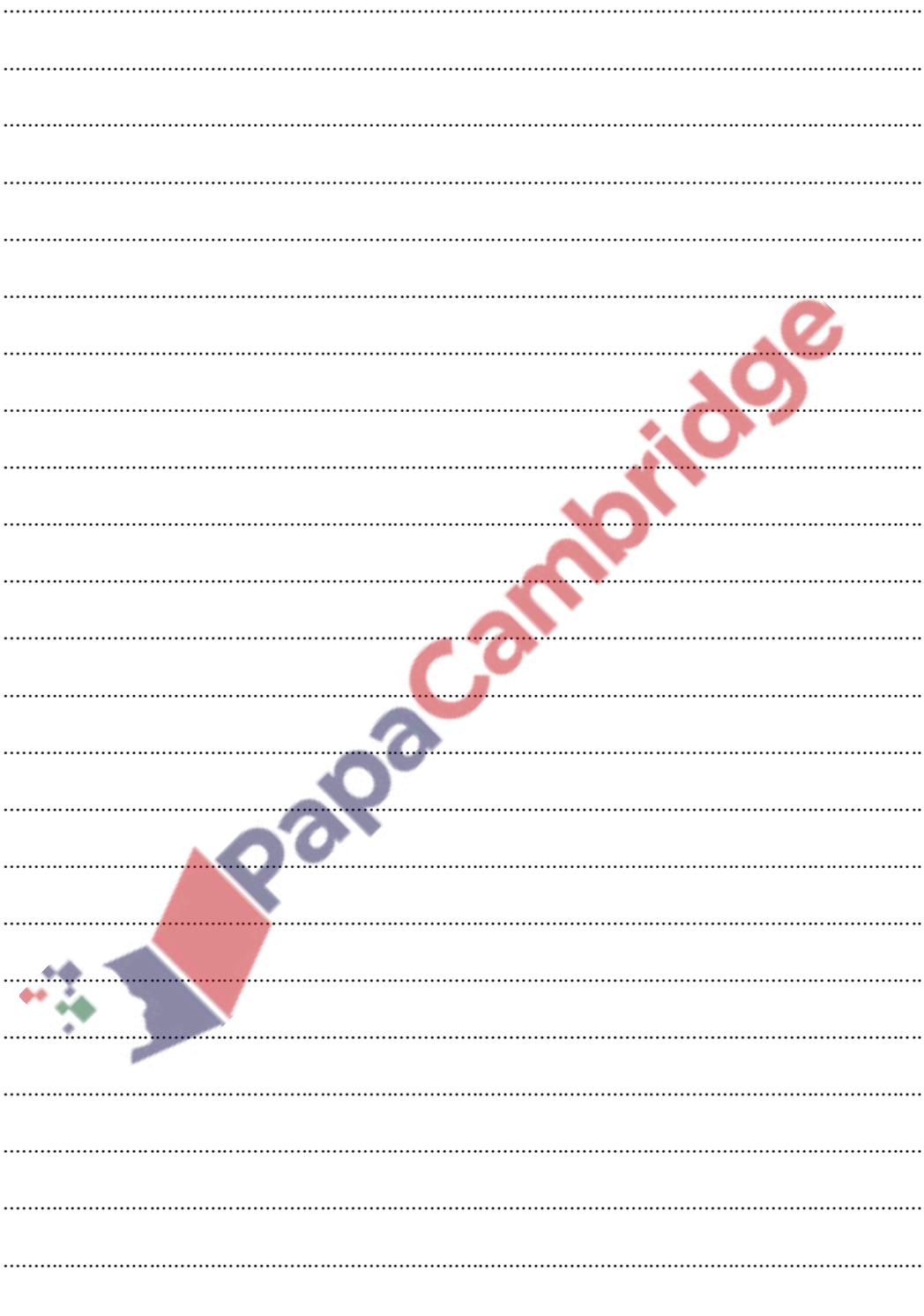
Chapter 5

Integration

146. 9709_s20_qp_21 Q: 6

(a) Prove that

$$\sin 2\theta (\operatorname{cosec} \theta - \sec \theta) \equiv \sqrt{8} \cos\left(\theta + \tfrac{1}{4}\pi\right).$$

[5]

(b) Solve the equation

$$\sin 2\theta(\operatorname{cosec} \theta - \sec \theta) = 1$$

for $0 < \theta < \frac{1}{2}\pi$. Give the answer correct to 3 significant figures. [2]

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(c) Find $\displaystyle\int \sin x\left(\operatorname{cosec} \tfrac{1}{2}x - \sec \tfrac{1}{2}x\right) dx.$ [3]

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147. 9709_s20_qp_21 Q: 7

(a) Find the quotient when $9x^3 - 6x^2 - 20x + 1$ is divided by $(3x + 2)$, and show that the remainder is 9. [3]

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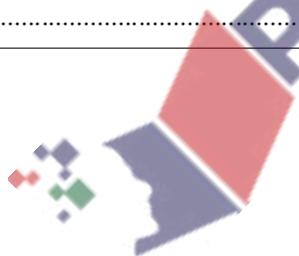
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(b) Hence find $\displaystyle\int_1^6 \frac{9x^3 - 6x^2 - 20x + 1}{3x + 2}\, dx$, giving the answer in the form $a + \ln b$ where a and b are integers. [5]
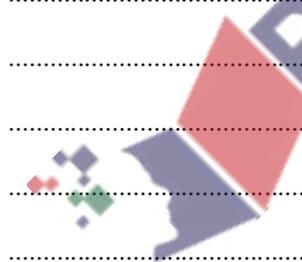
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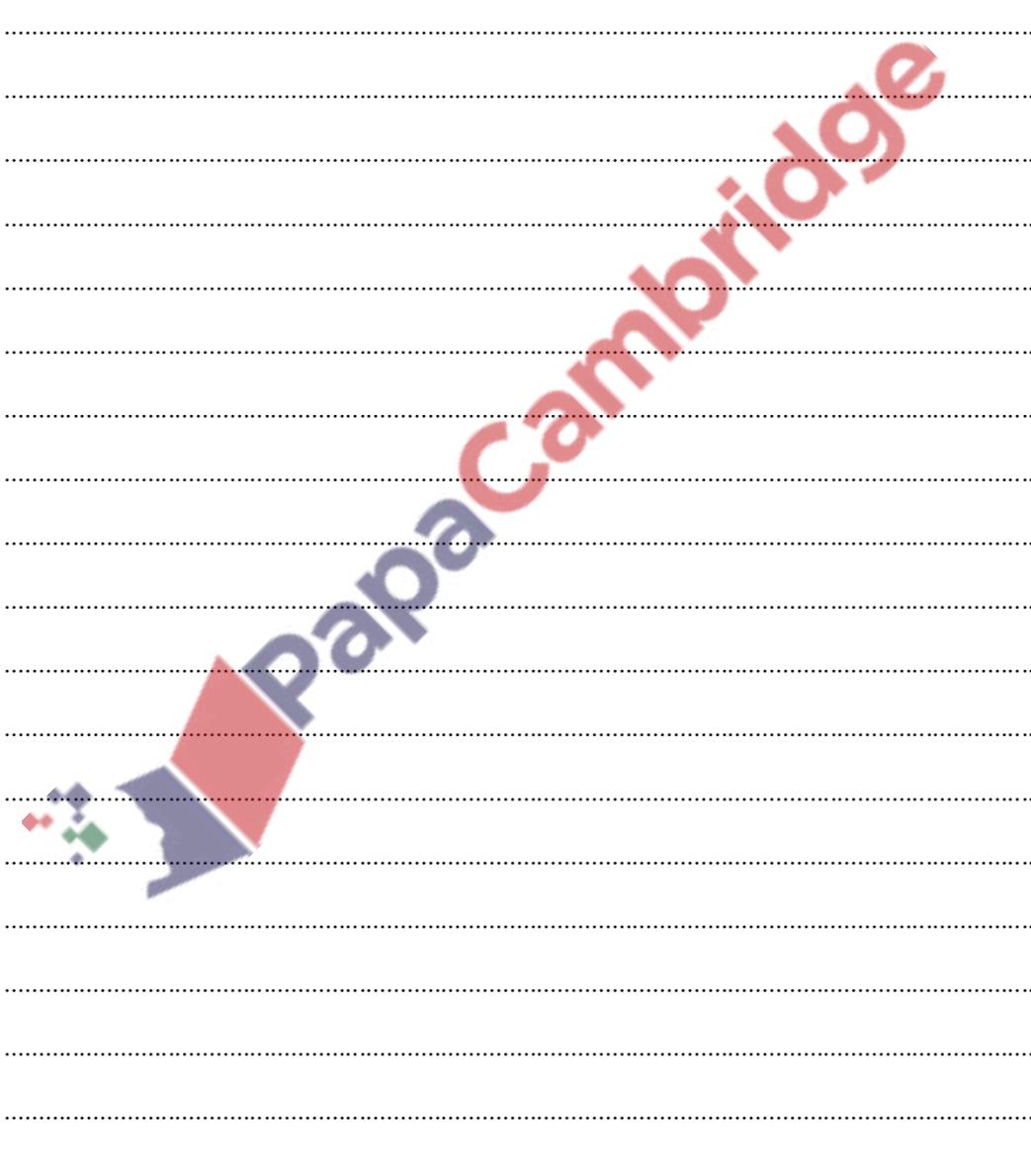
(c) Find the exact root of the equation $9e^{9y} - 6e^{6y} - 20e^{3y} - 8 = 0$. [4]

148. 9709_s20_qp_22 Q: 7

It is given that $\displaystyle\int_0^a \left(\frac{4}{2x+1} + 8x \right) \mathrm{d}x = 10$, where a is a positive constant.

(a) Show that $a = \sqrt{2.5 - 0.5\ln(2a+1)}$. [4]

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(b) Using the equation in part **(a)**, show by calculation that $1 < a < 2$. [2]

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(c) Use an iterative formula, based on the equation in part **(a)**, to find the value of a correct to 4 significant figures. Give the result of each iteration to 6 significant figures. [3]

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149. 9709_s20_qp_22 Q: 8

(a) Show that $3 \sin 2\theta \cot \theta \equiv 6 \cos^2 \theta$. [2]

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(b) Solve the equation $3 \sin 2\theta \cot \theta = 5$ for $0 < \theta < \pi$. [3]

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(c) Find the exact value of $\displaystyle\int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} 3\sin x \cot \tfrac{1}{2}x \, \mathrm{d}x.$ [5]

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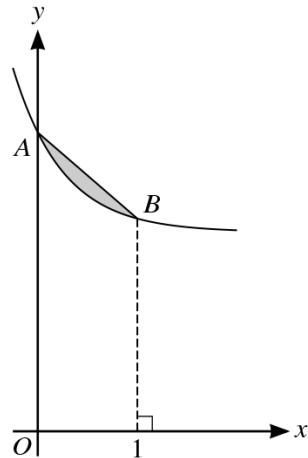
150. 9709_w20_qp_21 Q: 3

The diagram shows the curve $y = 2 + e^{-2x}$. The curve crosses the y-axis at the point A, and the point B on the curve has x-coordinate 1. The shaded region is bounded by the curve and the line segment AB.
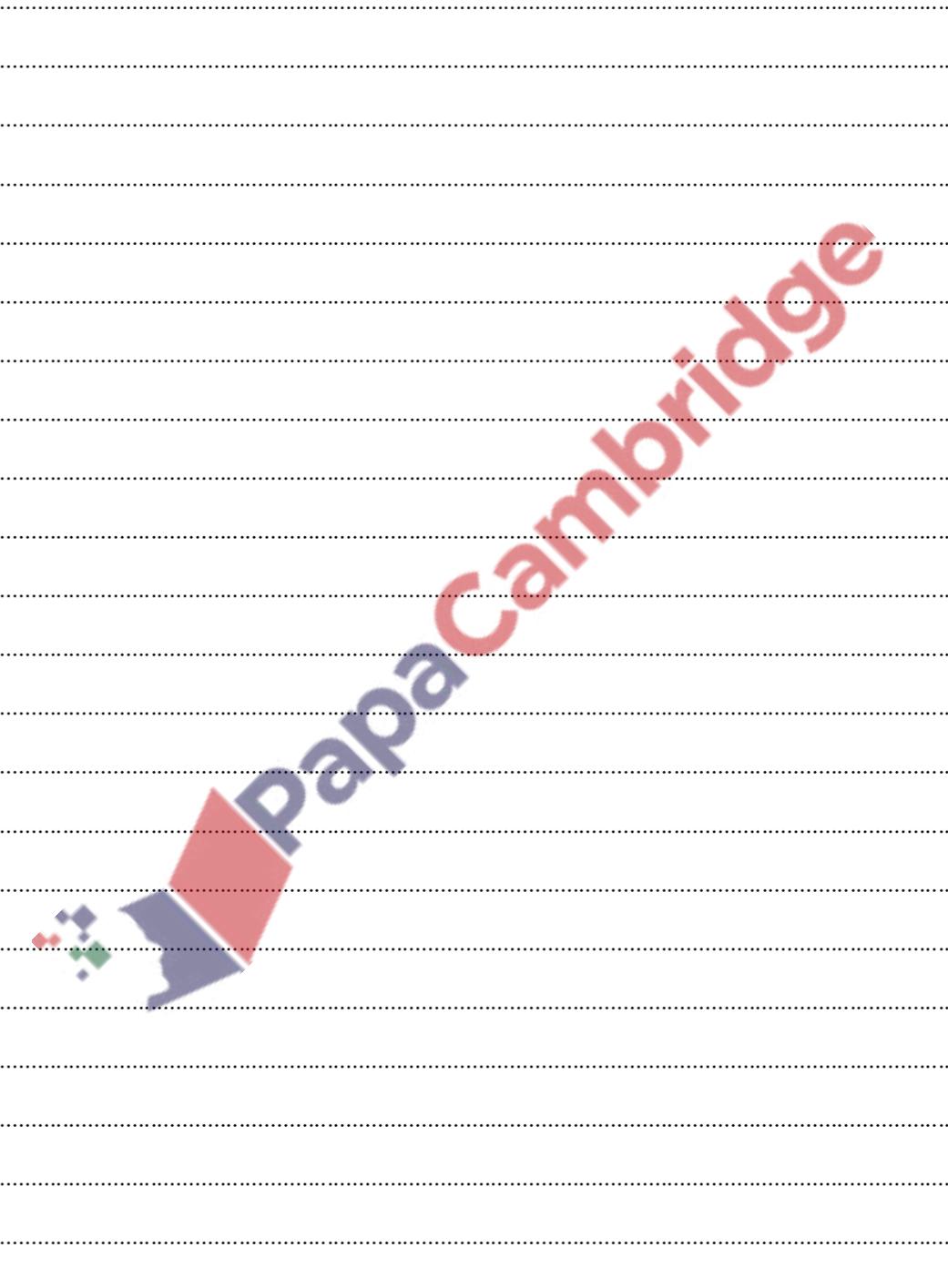
Find the exact area of the shaded region. [5]

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151. 9709_w20_qp_21   Q: 8

A curve has equation $y = f(x)$ where $f(x) = \dfrac{4x^3 + 8x - 4}{2x - 1}$.

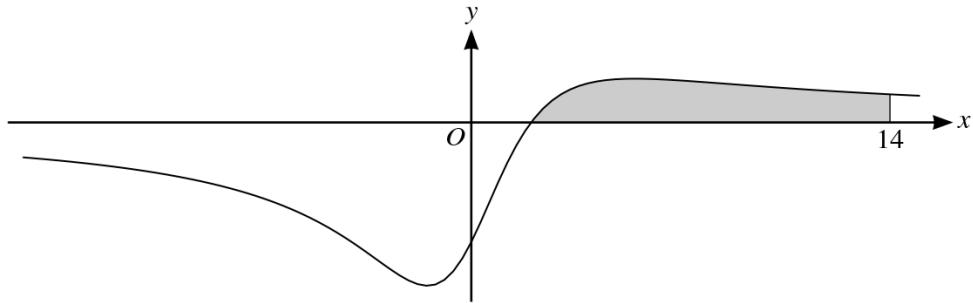**(a)** Find an expression for $\dfrac{dy}{dx}$ and hence find the coordinates of each of the stationary points of the curve $y = f(x)$. [5]

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(b) Divide $4x^3 + 8x - 4$ by $(2x - 1)$, and hence find $\int \text{f}(x)\,\text{d}x$. [5]

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152. 9709_w20_qp_22 Q: 4

The diagram shows the curve with equation $y = \dfrac{x-2}{x^2+8}$. The shaded region is bounded by the curve and the lines $x = 14$ and $y = 0$.
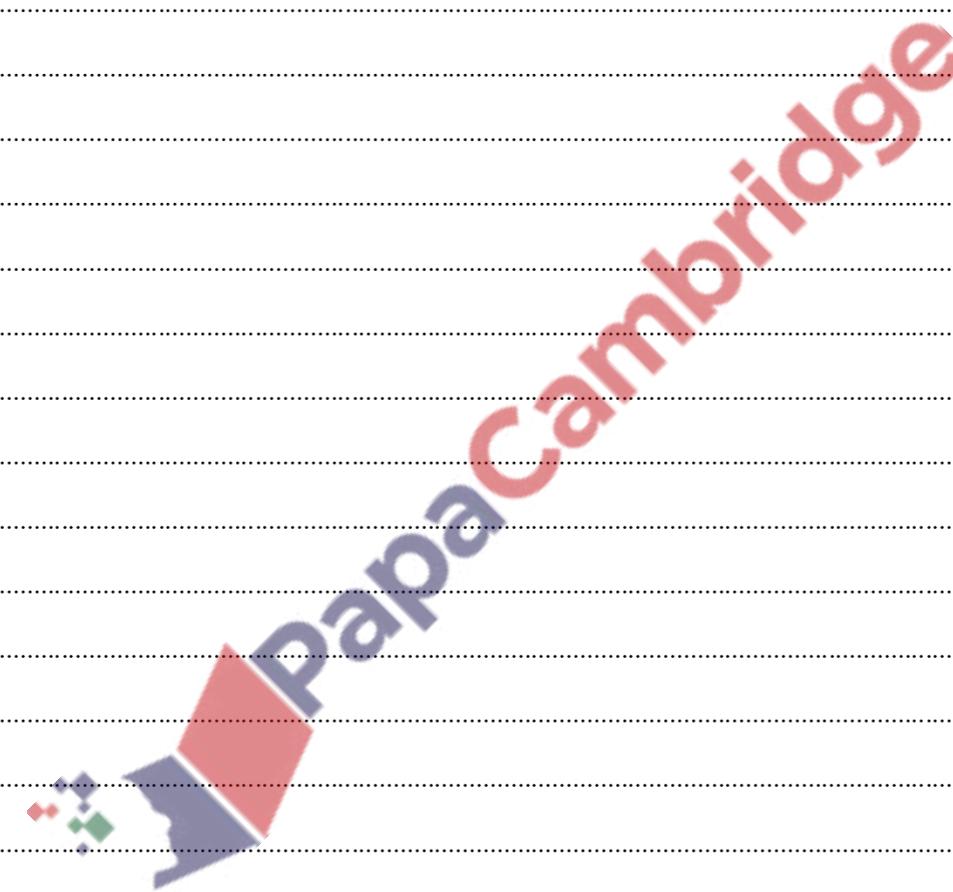
(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and hence determine the exact x-coordinates of the stationary points. [4]

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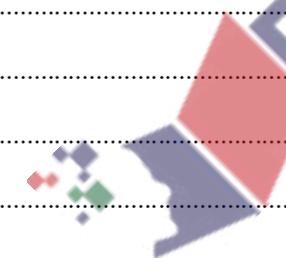
(b) Use the trapezium rule with three intervals to find an approximation to the area of the shaded
region. Give the answer correct to 2 significant figures.   [3]

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153. 9709_w20_qp_22 Q: 6

(a) Find $\displaystyle\int\left(\frac{8}{4x+1}+\frac{8}{\cos^2(4x+1)}\right)dx.$ [4]

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(b) It is given that $\int_0^{\frac{1}{2}\pi} (3 + 4\cos^2 \frac{1}{2}x + k\sin 2x)\,\mathrm{d}x = 10$.

Find the exact value of the constant k. [6]
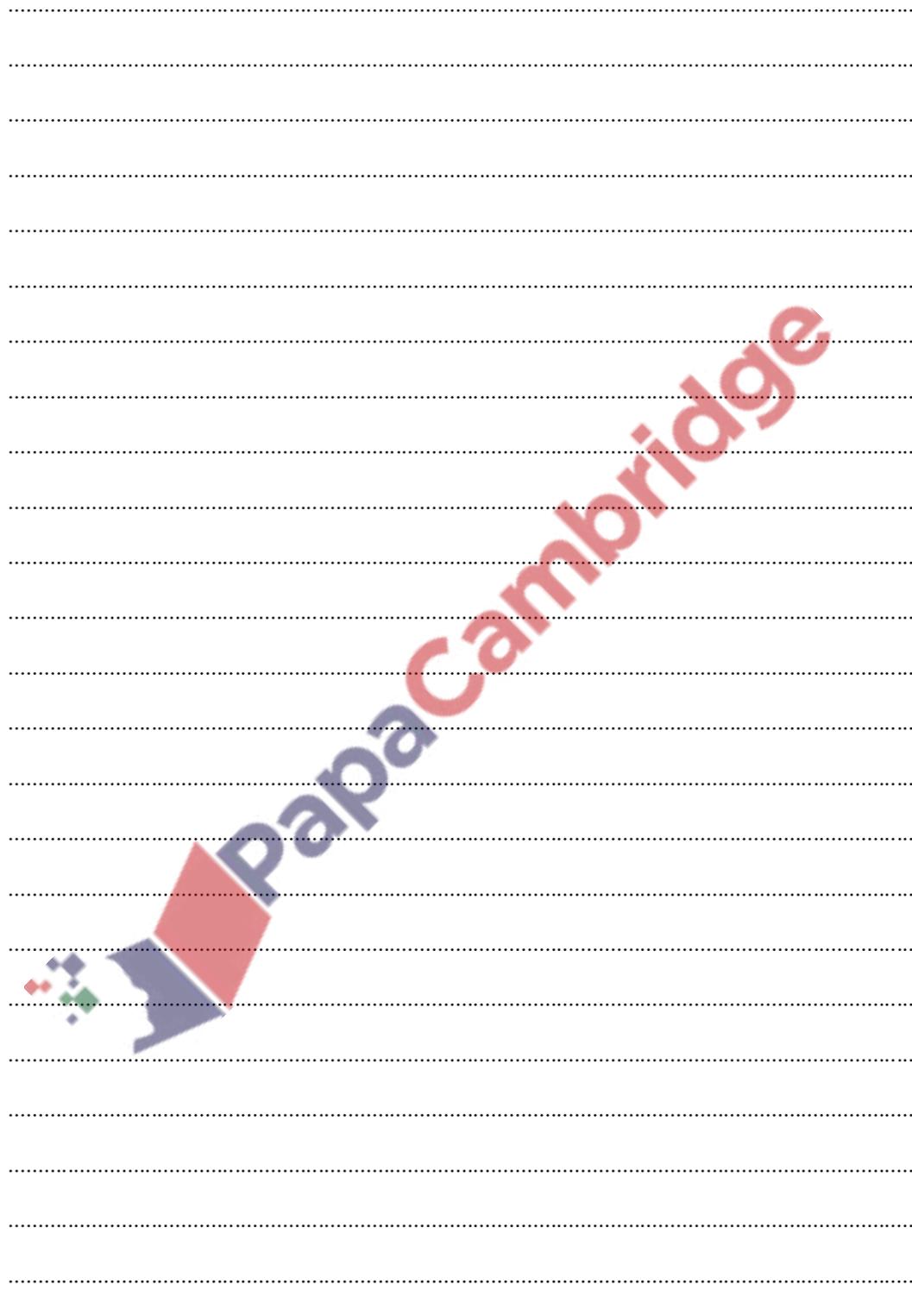
154. 9709_m19_qp_22 Q: 6

(a) Show that $\displaystyle\int_1^4 \left(\frac{2}{x} + \frac{2}{2x+1}\right) dx = \ln 48.$ [5]

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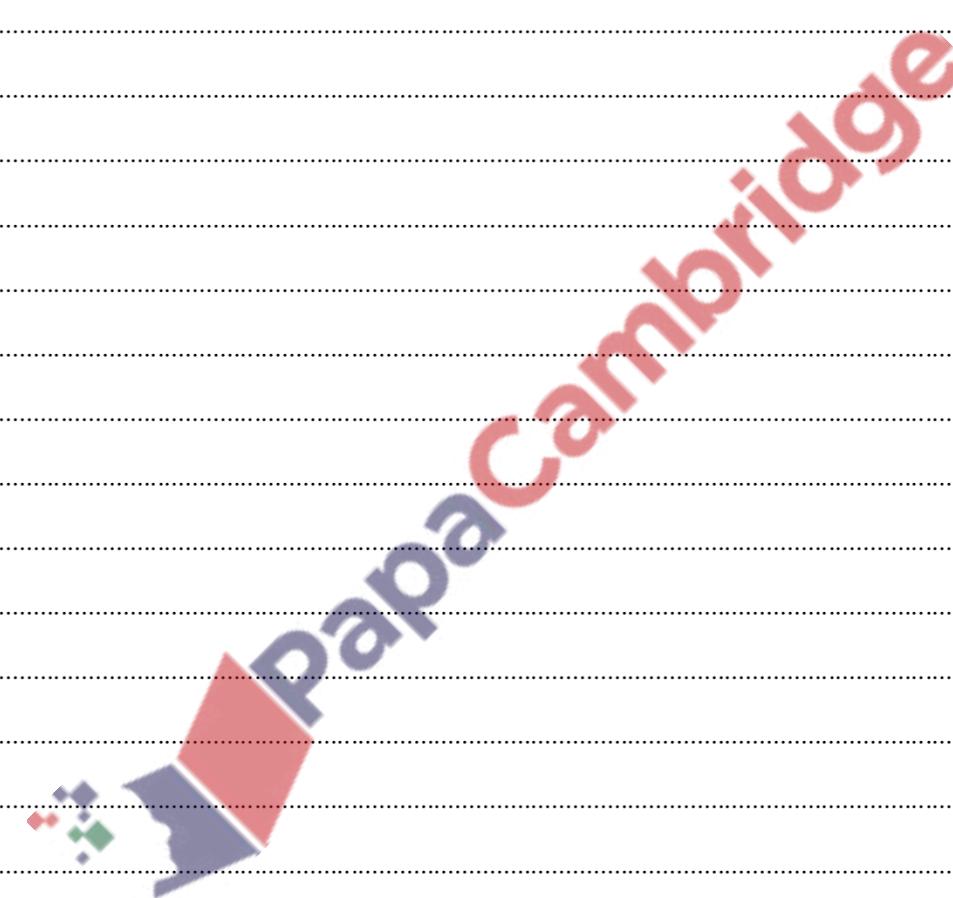
(b) Find $\int \sin 2x \, (\cot x + 2 \operatorname{cosec} x) \, dx.$ [6]

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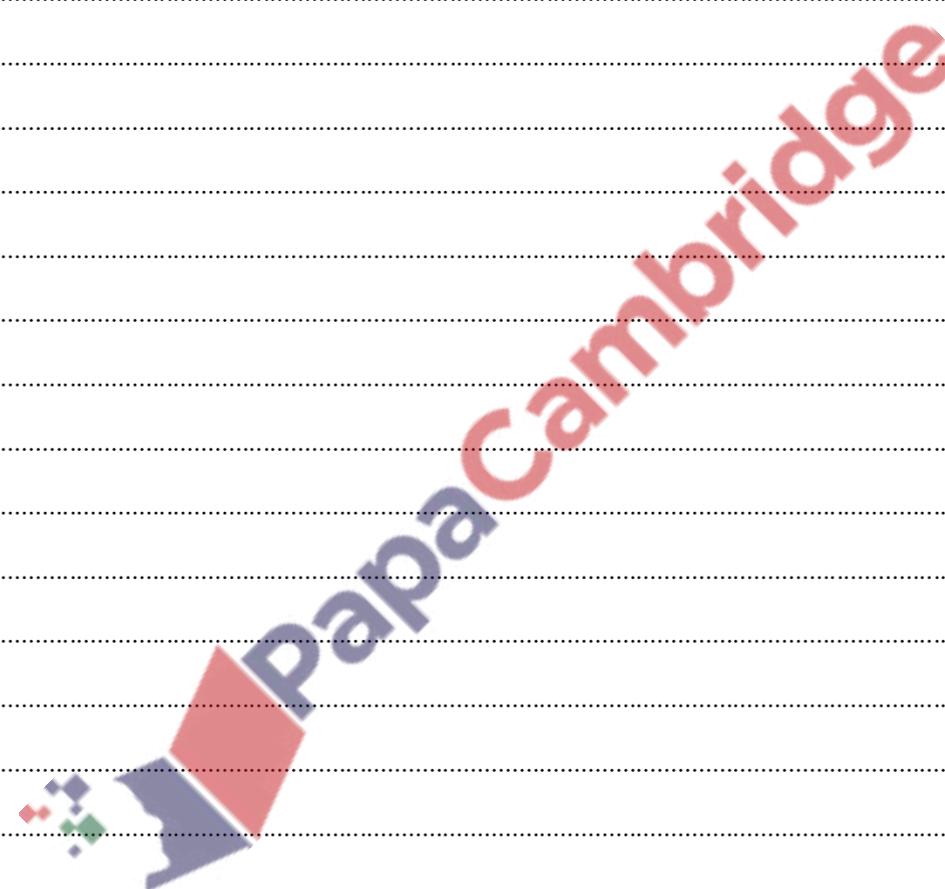
155. 9709_s19_qp_21 Q: 4

(a) Find $\int \tan^2 3x \, dx$. [3]

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(b) Find the exact value of $\displaystyle\int_0^1 \frac{e^{3x} + 4}{e^x} \, dx$. Show all necessary working. [4]

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156. 9709_s19_qp_22 Q: 4

(a) Find the exact value of $\int_0^{\frac{1}{2}\pi} (4\sin 2x + 2\cos^2 x)\,dx$. Show all necessary working. [5]

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(b) Use the trapezium rule with two intervals to find an approximation to $\int_2^8 \sqrt{(\ln(1+x))}\,dx$. [3]

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157. 9709_s19_qp_22 Q: 5

(i) Find the quotient and remainder when $2x^3 + x^2 - 8x$ is divided by $(2x + 1)$. [3]

(ii) Hence find the exact value of $\displaystyle\int_0^3 \frac{2x^3 + x^2 - 8x}{2x + 1}\,\mathrm{d}x$, giving the answer in the form $\ln(k\mathrm{e}^a)$ where k and a are constants. [5]

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158. 9709_w19_qp_21 Q: 2

Find the exact value of $\int_1^2 (2e^{2x} - 1)^2 \, dx$. Show all necessary working. [5]

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159. 9709_w19_qp_22 Q: 6

(a) Show that $\displaystyle\int_2^{18} \frac{3}{2x}\,dx = \ln 27.$ [4]

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(b) Find the exact value of $\displaystyle\int_0^{\frac{1}{6}\pi} 4\sin^2\!\left(\tfrac{3}{2}x\right) dx.$ Show all necessary working. [5]

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160. 9709_m18_qp_22 Q: 3

(i) Use the trapezium rule with four intervals to find an approximation to

$$\int_0^8 \ln(x + 2)\, dx,$$

giving your answer correct to 3 significant figures. [3]

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(ii) Hence find an approximation to

$$\int_0^8 3\ln(x^2 + 4x + 4)\, dx.$$ [2]

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161. 9709_m18_qp_22 Q: 6

(i) Show that $\operatorname{cosec} 2x + \cot 2x \equiv \cot x$. [4]

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(ii) Hence find the exact value of $\cot \frac{1}{12}\pi$. [2]

(iii) Find $\displaystyle\int \sin 2x(\operatorname{cosec} 4x + \cot 4x)\,dx$. [3]

162. 9709_s18_qp_21 Q: 3

Without using a calculator, find the exact value of $\int_0^2 4e^{-x}(e^{3x} + 1)\,dx.$ [5]

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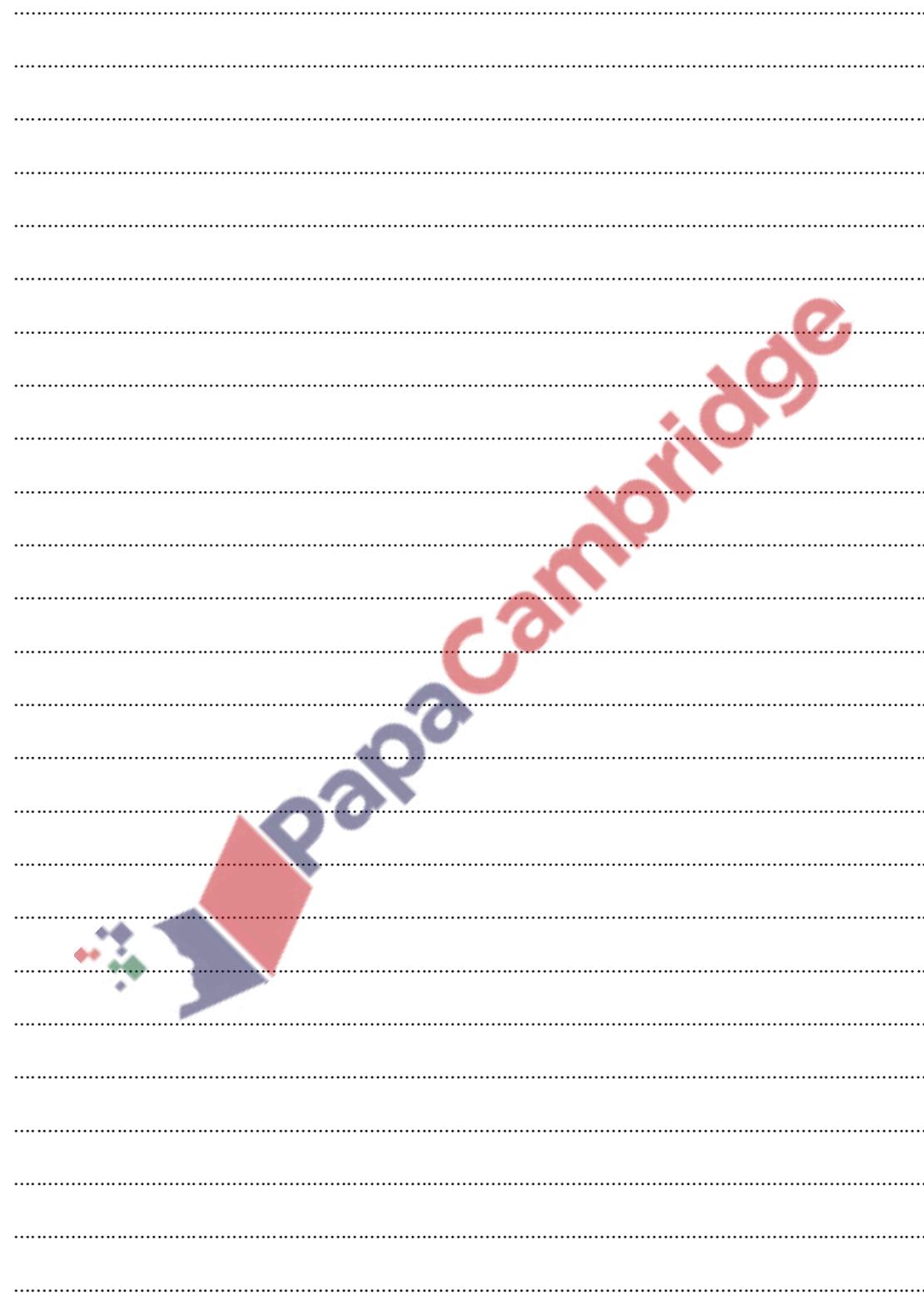
163. 9709_s18_qp_21 Q: 7

(i) Express $5\cos\theta - 2\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Give the value of α correct to 4 decimal places. [3]

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(ii) Using your answer from part (i), solve the equation

$$5\cot\theta - 4\operatorname{cosec}\theta = 2$$

for $0 < \theta < 2\pi$. [5]

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(iii) Find $\displaystyle\int \frac{1}{\left(5\cos\frac{1}{2}x - 2\sin\frac{1}{2}x\right)^2}\,\mathrm{d}x.$ [3]

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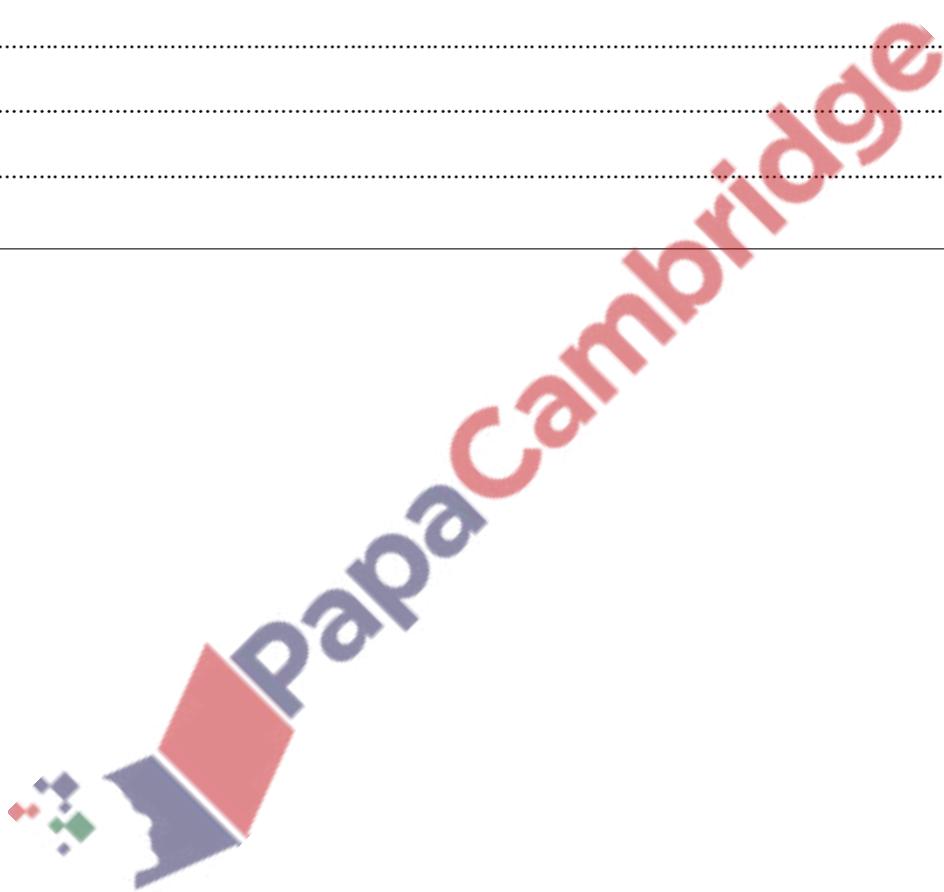
164. 9709_s18_qp_22   Q: 7

(i) Show that $2 \operatorname{cosec}^2 2x(1 - \cos 2x) \equiv \sec^2 x$. [3]

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(ii) Solve the equation $2 \operatorname{cosec}^2 2x(1 - \cos 2x) = \tan x + 21$ for $0 < x < \pi$, giving your answers correct to 3 significant figures. [4]

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(iii) Find $\int [2\operatorname{cosec}^2(4y+2) - 2\operatorname{cosec}^2(4y+2)\cos(4y+2)]\,dy.$ [3]

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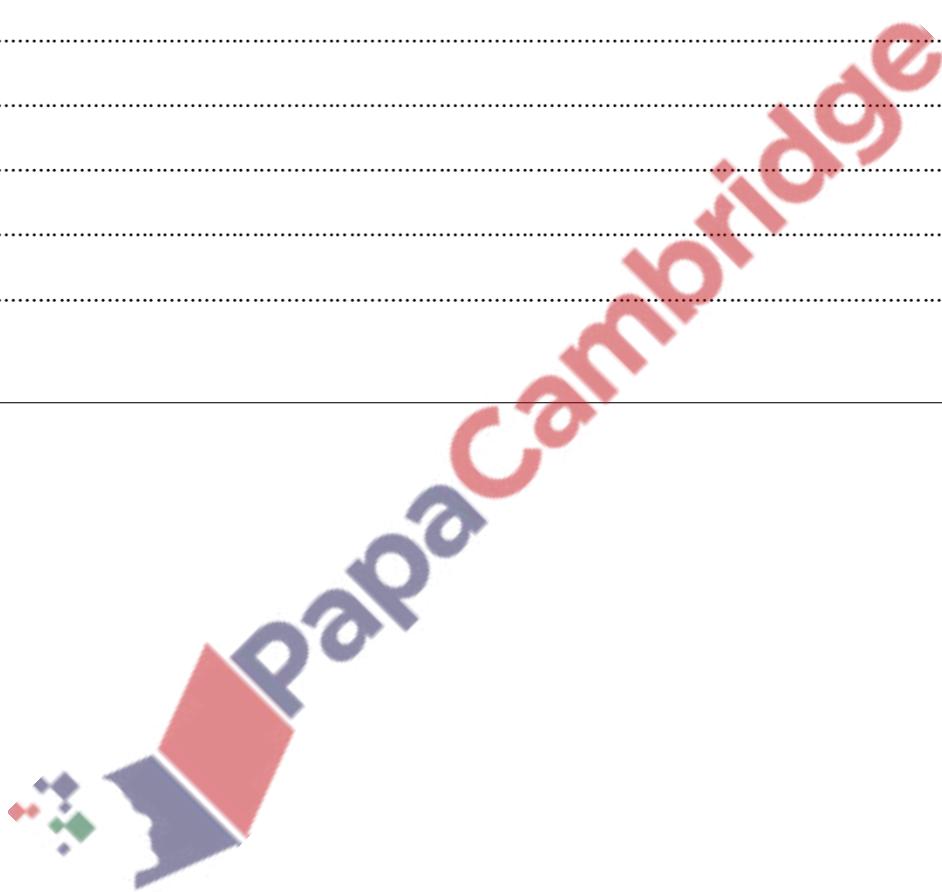
165. 9709_w18_qp_21 Q: 2

Show that $\displaystyle\int_{1}^{7} \frac{6}{2x+1}\,\mathrm{d}x = \ln 125.$

[5]
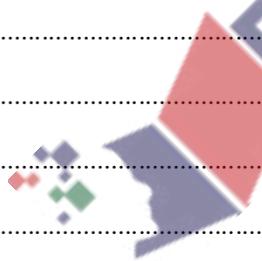
166. 9709_w18_qp_21 Q: 6
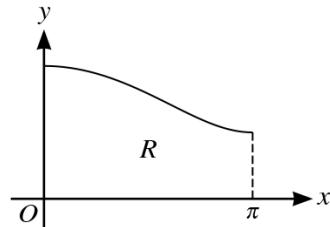
The diagram shows the curve with equation $y = \sqrt{\left(1 + 3\cos^2\left(\frac{1}{2}x\right)\right)}$ for $0 \leqslant x \leqslant \pi$. The region R is bounded by the curve, the axes and the line $x = \pi$.

(i) Use the trapezium rule with two intervals to find an approximation to the area of R, giving your answer correct to 3 significant figures. [3]
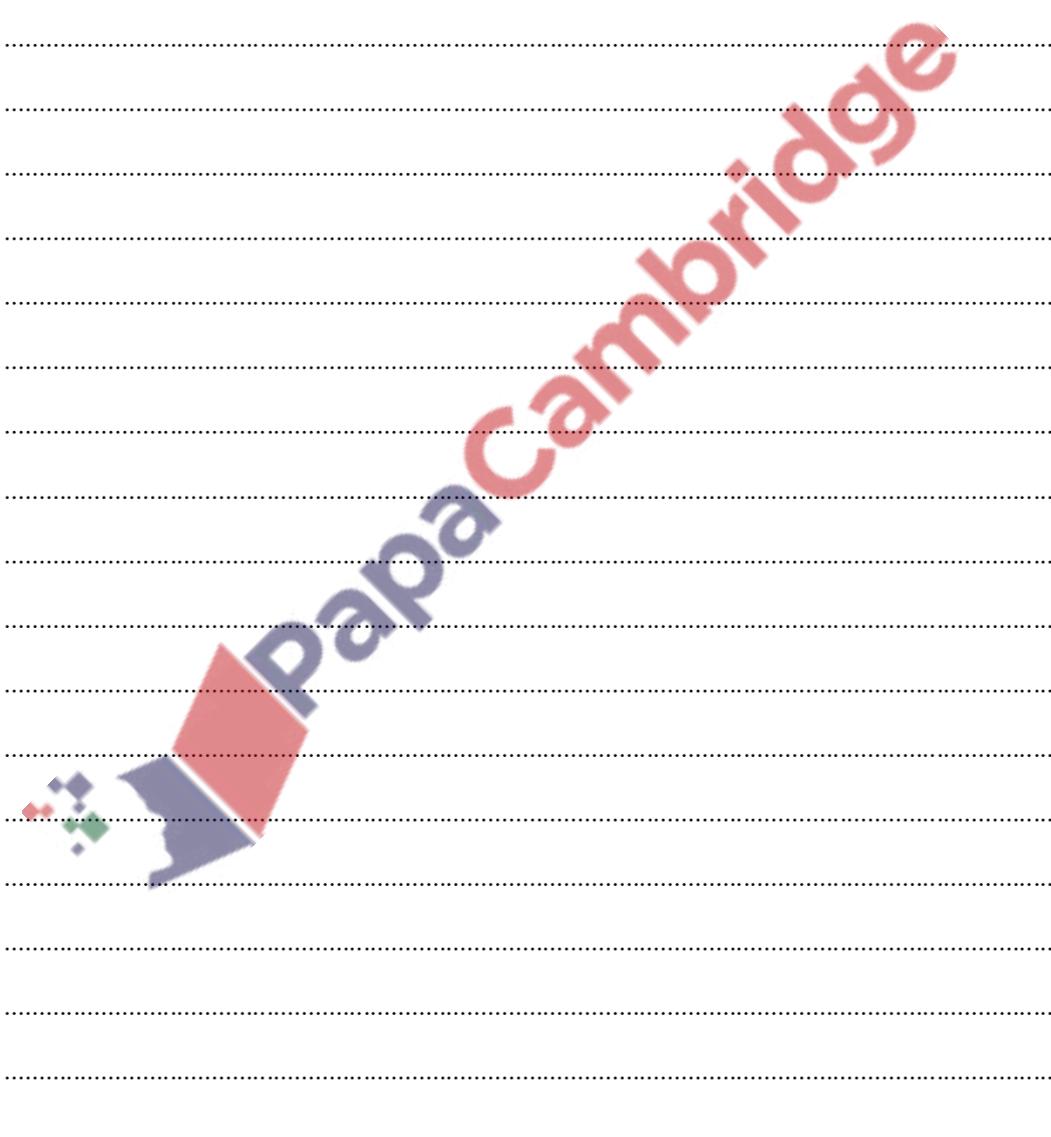
(ii) The region R is rotated completely about the x-axis. Without using a calculator, find the exact volume of the solid produced. [5]

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167. 9709_w18_qp_22 Q: 6

(a) Show that $\displaystyle\int_1^6 \frac{12}{3x+2}\,dx = \ln 256.$ [5]
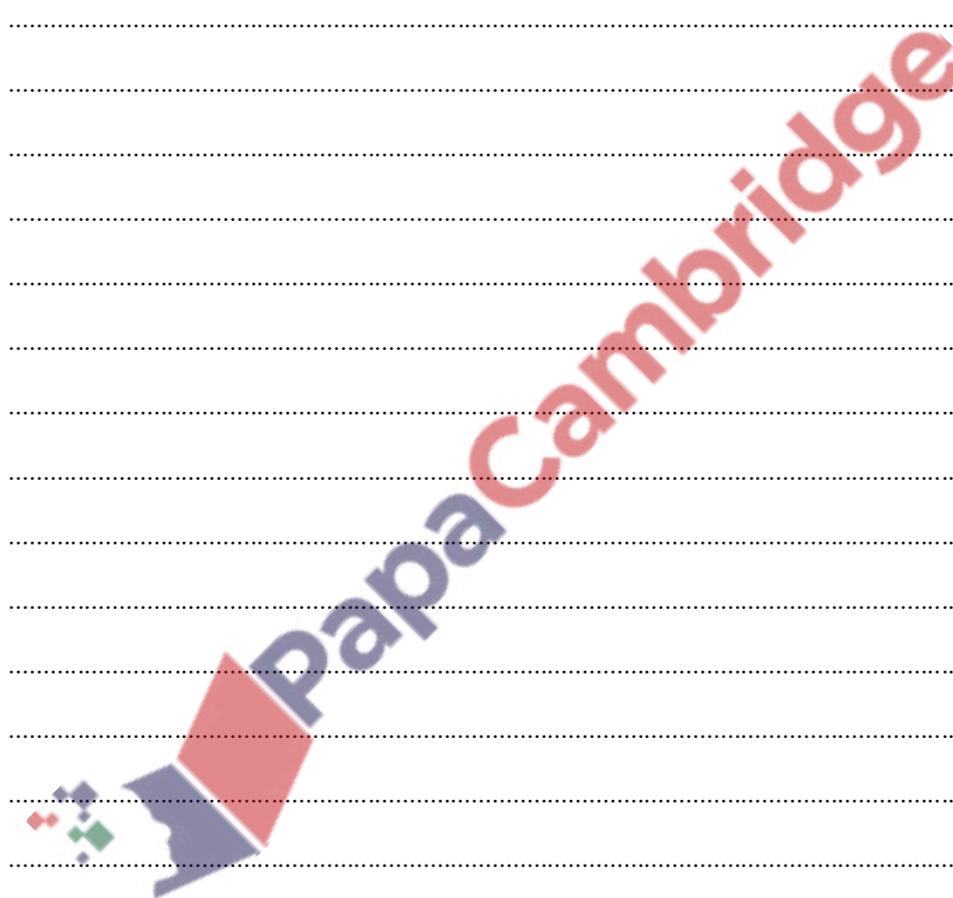
(b) Find the exact value of $\displaystyle\int_0^{\frac{1}{6}\pi} (8\sin^2 x + \tan^2 2x)\,\mathrm{d}x$, showing all necessary working. [6]

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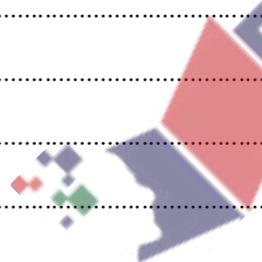
168. 9709_m17_qp_22 Q: 7
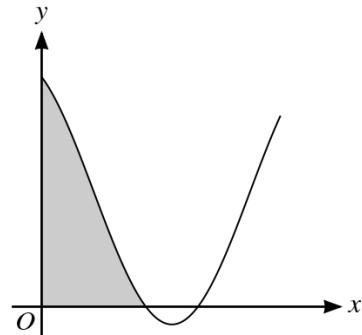
The diagram shows part of the curve

$$y = 2 \cos 2x \cos\left(2x + \tfrac{1}{6}\pi\right).$$

The shaded region is bounded by the curve and the two axes.

(i) Show that $2 \cos 2x \cos\left(2x + \tfrac{1}{6}\pi\right)$ can be expressed in the form

$$k_1(1 + \cos 4x) + k_2 \sin 4x,$$

where the values of the constants k_1 and k_2 are to be determined. [5]

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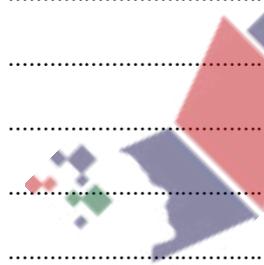
(ii) Find the exact area of the shaded region. [5]

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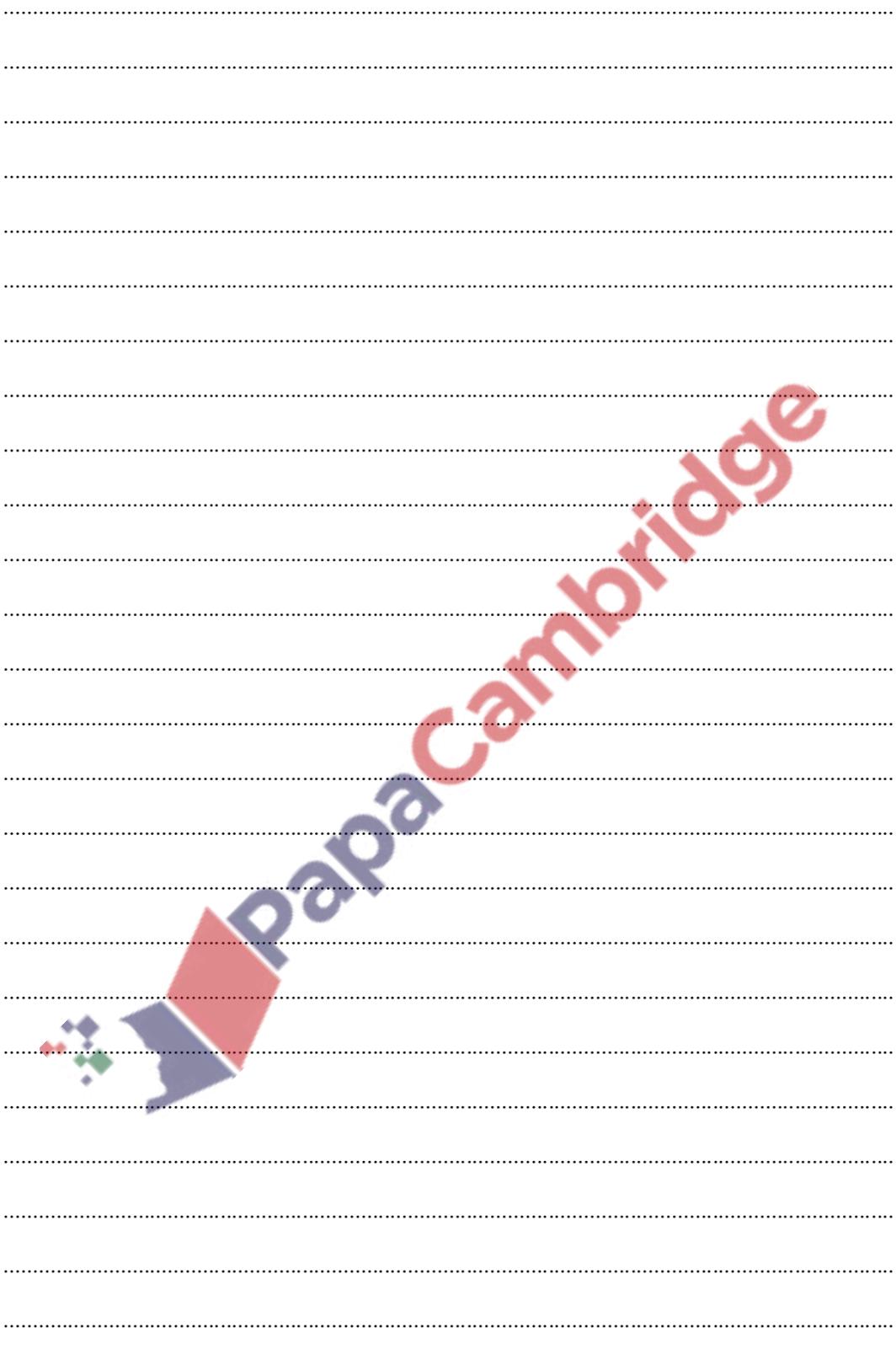
169. 9709_s17_qp_21   Q: 3

Given that $\displaystyle\int_0^a 4e^{\frac{1}{2}x+3}\,dx = 835$, find the value of the constant a correct to 3 significant figures.   [5]

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170. 9709_s17_qp_21 Q: 6
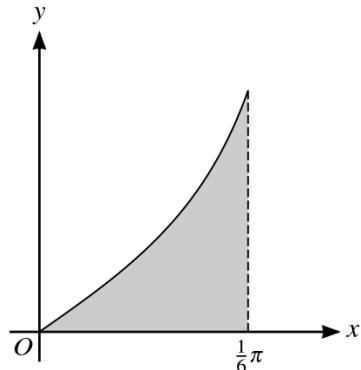
The diagram shows the curve $y = \tan 2x$ for $0 \leqslant x \leqslant \frac{1}{6}\pi$. The shaded region is bounded by the curve and the lines $x = \frac{1}{6}\pi$ and $y = 0$.

(i) Use the trapezium rule with two intervals to find an approximation to the area of the shaded region, giving your answer correct to 3 significant figures. [3]

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(ii) Find the exact volume of the solid formed when the shaded region is rotated completely about the x-axis. [4]
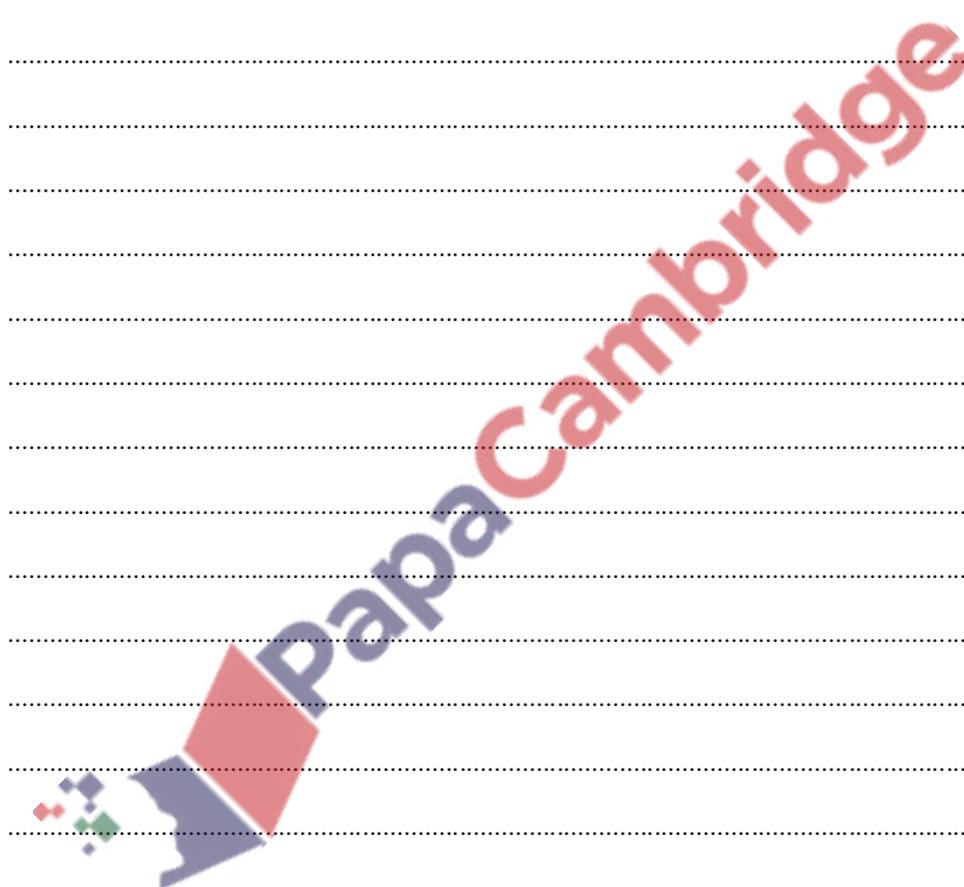
171. 9709_s17_qp_22   Q: 7

(a) Find $\int (2\cos\theta - 3)(\cos\theta + 1)\, d\theta$.   [4]

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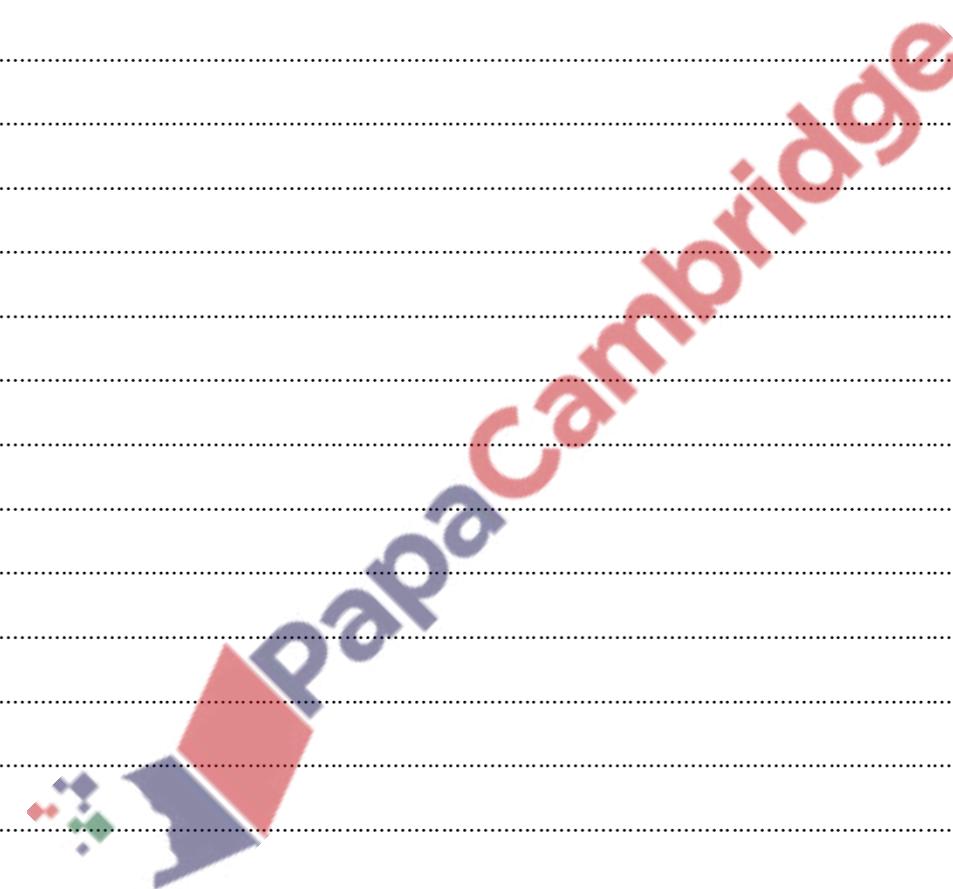
(b) **(i)** Find $\int \left(\dfrac{4}{2x+1} + \dfrac{1}{2x} \right) dx.$ [2]

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(ii) Hence find $\int_1^4 \left(\dfrac{4}{2x+1} + \dfrac{1}{2x} \right) dx,$ giving your answer in the form $\ln k.$ [3]

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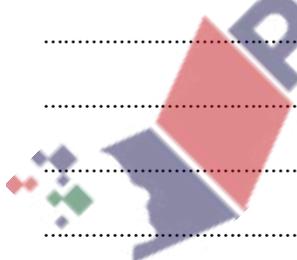
172. 9709_w17_qp_21   Q: 4

(a) Find $\displaystyle\int \frac{4 + \sin^2\theta}{1 - \sin^2\theta}\,d\theta.$ [4]

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(b) Given that $\displaystyle\int_0^a \frac{2}{3x + 1}\,dx = \ln 16$, find the value of the positive constant a. [4]

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173. 9709_w17_qp_22 Q: 6

(a) Find the exact value of $\displaystyle\int_0^{\frac{1}{4}\pi} \sin x(4\sin x + 6\cos x)\,dx.$ [5]

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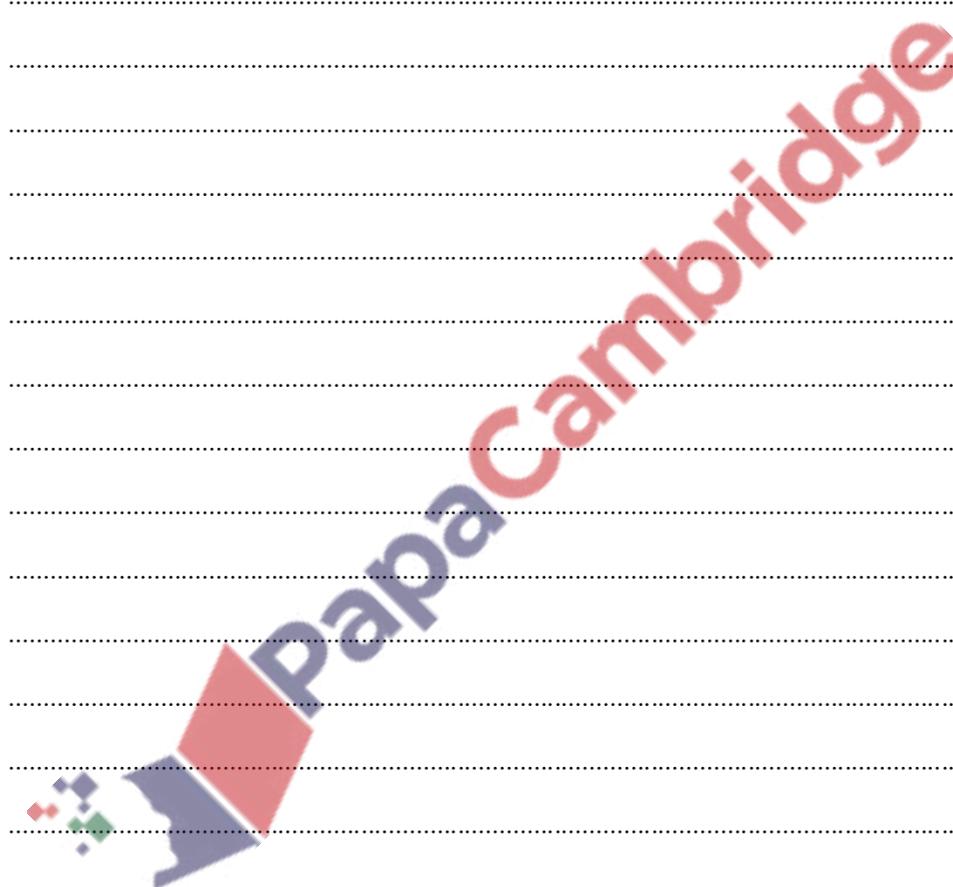
(b) Given that $\displaystyle\int_0^a \frac{6}{3x+2}\, dx = \ln 49$, find the value of the positive constant a. [5]
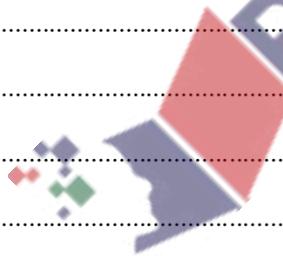
174. 9709_m16_qp_22 Q: 5

Given that $\int_0^a 6e^{2x+1}\,dx = 65$, find the value of a correct to 3 decimal places. [5]
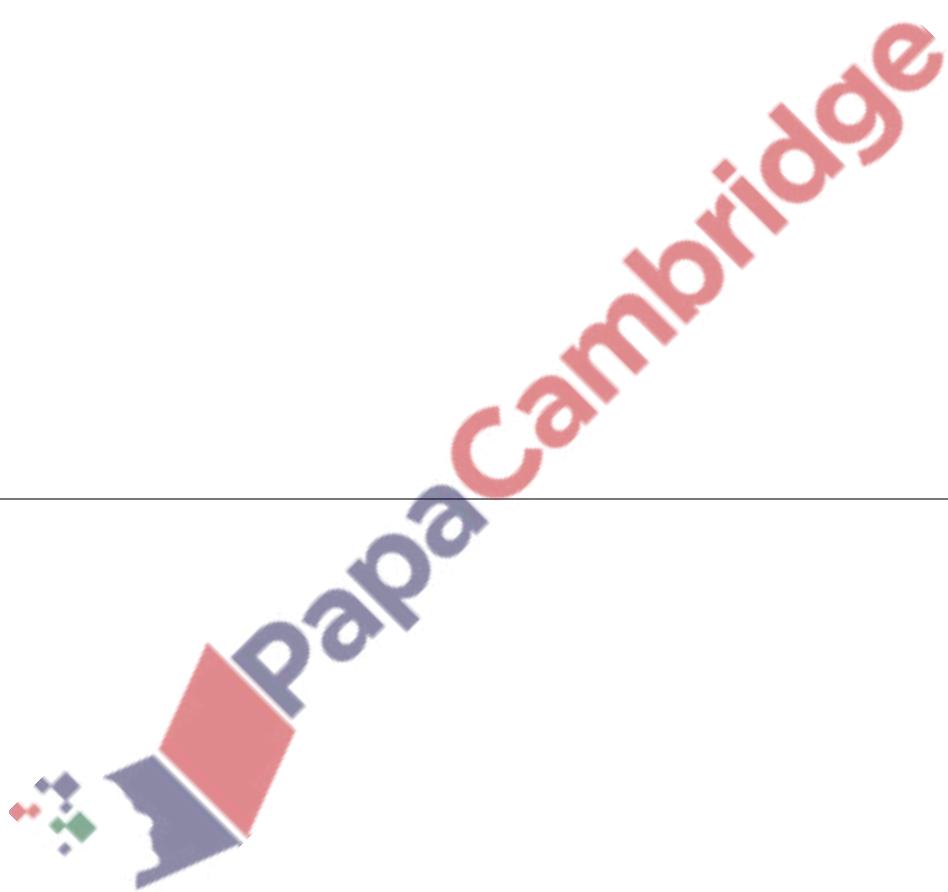
175. 9709_m16_qp_22 Q: 8

   (i) Show that $\sin 2x \cot x \equiv 2\cos^2 x$.   [2]

   (ii) Using the identity in part **(i)**,

        (a) find the least possible value of

$$3 \sin 2x \cot x + 5 \cos 2x + 8$$

            as x varies,   [4]

        (b) find the exact value of $\displaystyle\int_{\frac{1}{8}\pi}^{\frac{1}{6}\pi} \operatorname{cosec} 4x \tan 2x \, dx$.   [5]

176. 9709_s16_qp_21   Q: 7

(a) Find $\displaystyle\int \frac{1 + \cos^4 2x}{\cos^2 2x}\, \mathrm{d}x.$ [5]

(b) Without using a calculator, find the exact value of $\displaystyle\int_4^{14} \left(2 + \frac{6}{3x - 2}\right) \mathrm{d}x$, giving your answer in the form $\ln(a\mathrm{e}^b)$, where a and b are integers. [5]
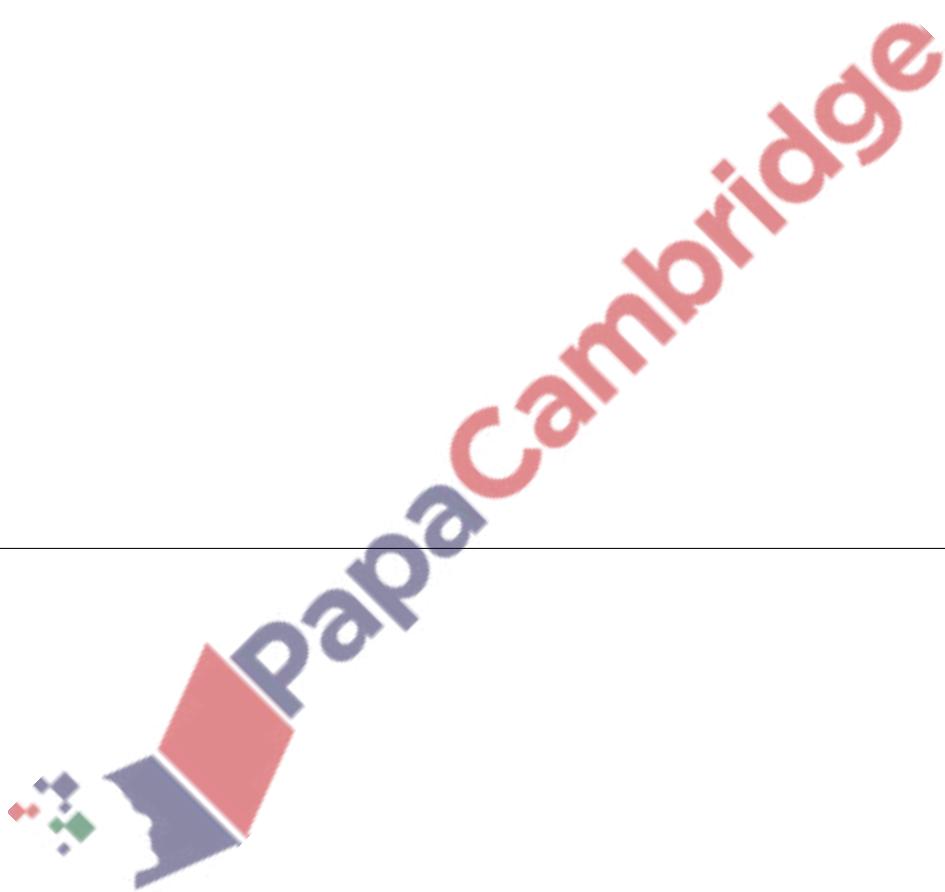
177. 9709_s16_qp_22   Q: 6

(a) Find $\displaystyle\int \frac{4 + e^x}{2e^{2x}}\, dx.$ [3]

(b) Without using a calculator, find $\displaystyle\int_2^{10} \frac{1}{2x + 5}\, dx$, giving your answer in the form $\ln k$. [4]

(c)
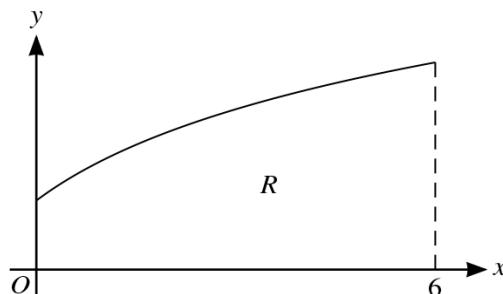
The diagram shows the curve $y = \log_{10}(x + 2)$ for $0 \leqslant x \leqslant 6$. The region bounded by the curve and the lines $x = 0$, $x = 6$ and $y = 0$ is denoted by R. Use the trapezium rule with 2 strips to find an estimate of the area of R, giving your answer correct to 1 decimal place. [3]

178. 9709_w16_qp_21   Q: 5

(i) Show that $\dfrac{\cos 2x + 9\cos x + 5}{\cos x + 4} \equiv 2\cos x + 1.$ [3]

(ii) Hence find the exact value of $\displaystyle\int_{-\pi}^{\pi} \dfrac{\cos 4x + 9\cos 2x + 5}{\cos 2x + 4}\,\mathrm{d}x.$ [4]

235
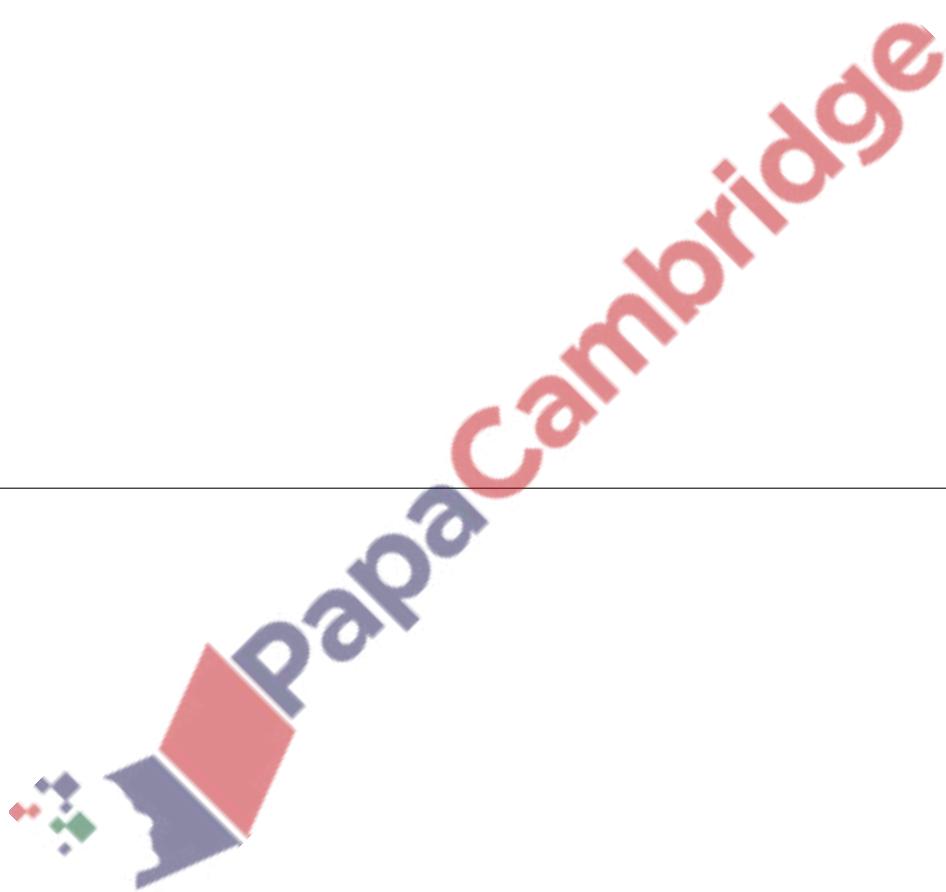
179. 9709_w16_qp_22 Q: 3

The definite integral I is defined by $I = \displaystyle\int_0^2 (4e^{\frac{1}{2}x} + 3)\,dx$.

(i) Show that $I = 8e - 2$. [3]

(ii) Sketch the curve $y = 4e^{\frac{1}{2}x} + 3$ for $0 \leqslant x \leqslant 2$. [2]

(iii) State whether an estimate of I obtained by using the trapezium rule will be more than or less than $8e - 2$. Justify your answer. [1]
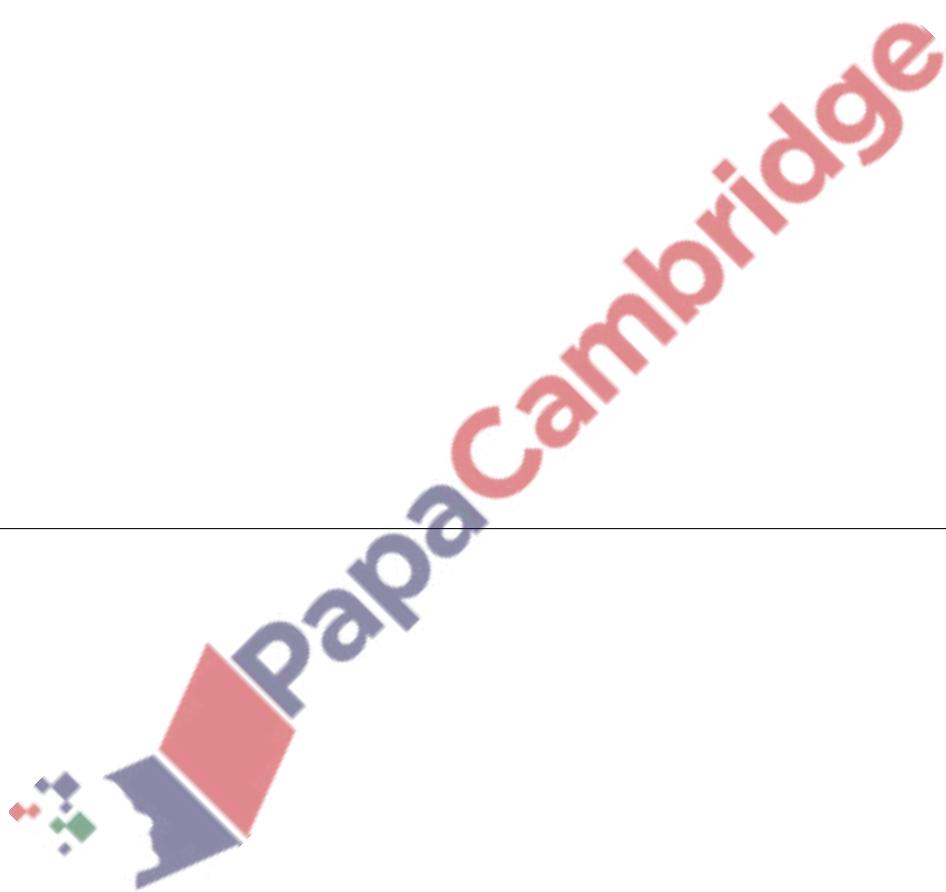
180. 9709_w16_qp_22 Q: 6

(i) Show that $\dfrac{\cos 2\theta}{1 + \cos 2\theta} \equiv 1 - \frac{1}{2}\sec^2\theta$. [2]

(ii) Solve the equation $\dfrac{\cos 2\alpha}{1 + \cos 2\alpha} = 13 + 5\tan\alpha$ for $0 < \alpha < \pi$. [4]

(iii) Find the exact value of $\displaystyle\int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} \dfrac{\cos x}{1 + \cos x}\,\mathrm{d}x$. [4]
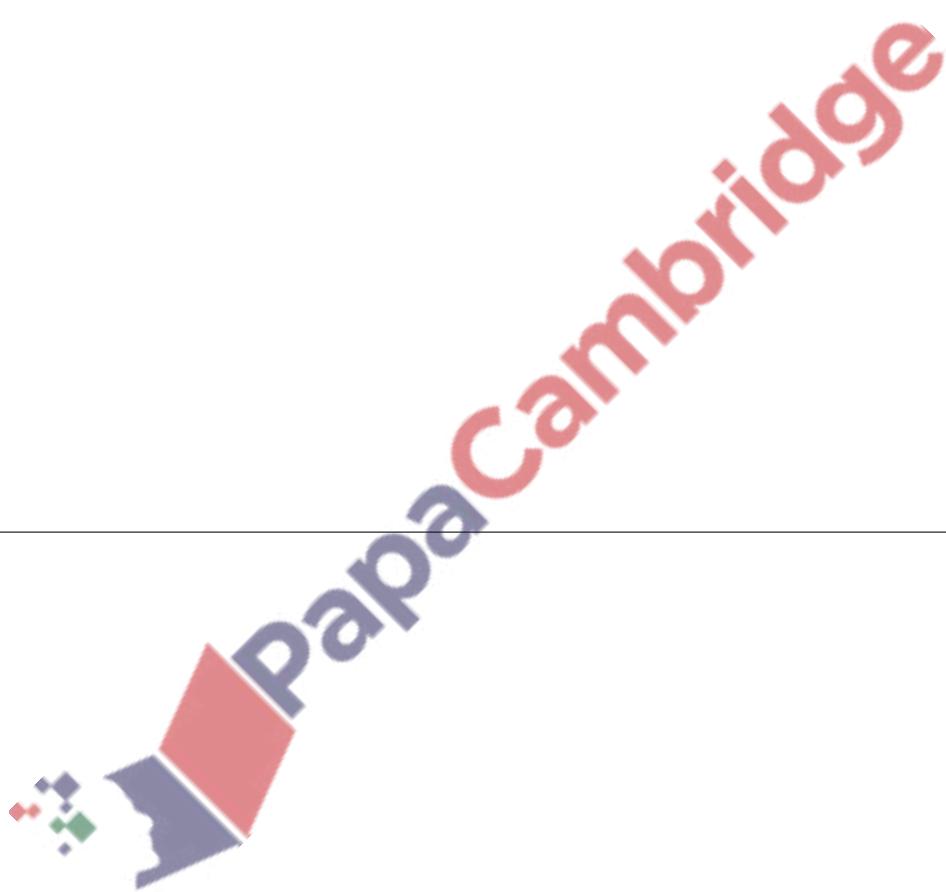
181. 9709_w16_qp_23 Q: 3

(i) Find $\int \tan^2 4x \, dx$. [2]

(ii) Without using a calculator, find the exact value of $\int_0^{\frac{1}{12}\pi} (4\cos 2x + 6\sin 3x) \, dx$. [3]
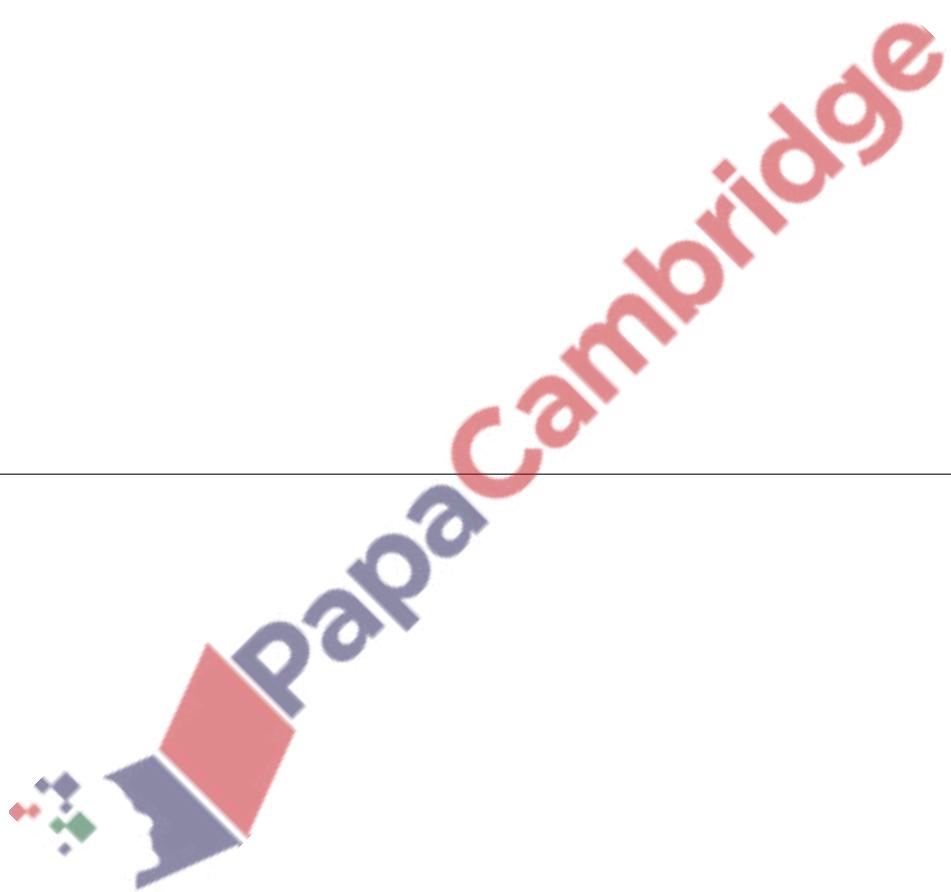
182. 9709_w16_qp_23 Q: 5
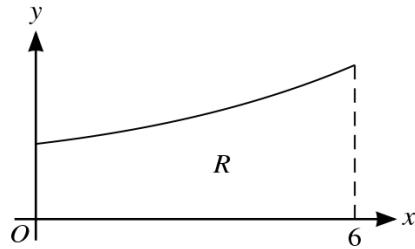
The diagram shows the curve $y = \sqrt{\left(1 + e^{\frac{1}{3}x}\right)}$ for $0 \leqslant x \leqslant 6$. The region bounded by the curve and the lines $x = 0$, $x = 6$ and $y = 0$ is denoted by R.

  (i) Use the trapezium rule with 2 strips to find an estimate of the area of R, giving your answer correct to 2 decimal places. [3]

 (ii) With reference to the diagram, explain why this estimate is greater than the exact area of R. [1]

 (iii) The region R is rotated completely about the x-axis. Find the exact volume of the solid produced. [4]

183. 9709_s15_qp_21 Q: 6

   (i) Prove that $2 \operatorname{cosec} 2\theta \tan \theta \equiv \sec^2 \theta$.   [3]

   (ii) Hence

       (a) solve the equation $2 \operatorname{cosec} 2\theta \tan \theta = 5$ for $0 < \theta < \pi$,   [3]

       (b) find the exact value of $\displaystyle\int_{0}^{\frac{1}{6}\pi} 2 \operatorname{cosec} 4x \tan 2x \, dx$.   [4]
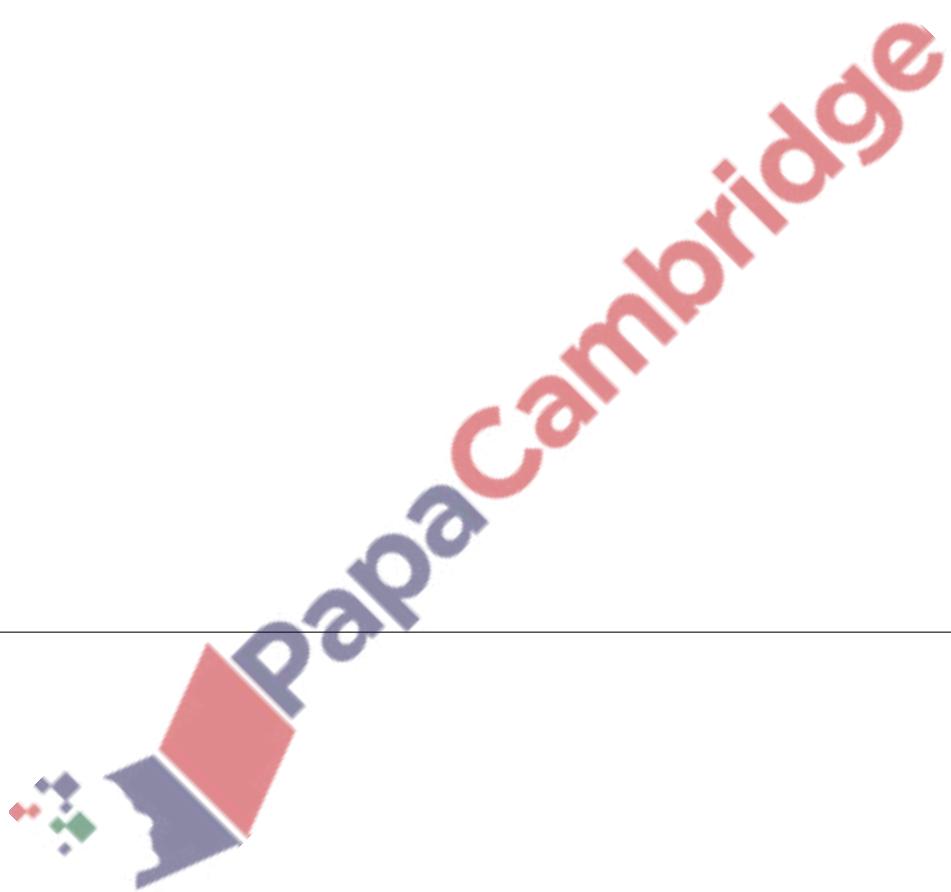
184. 9709_s15_qp_22 Q: 4
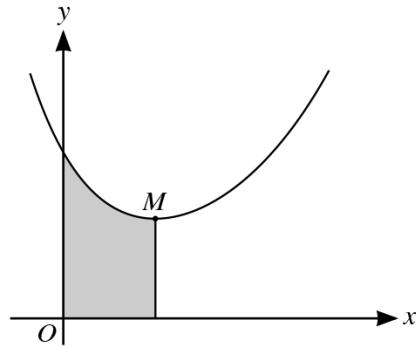
The diagram shows the curve $y = e^x + 4e^{-2x}$ and its minimum point M.

(i) Show that the x-coordinate of M is $\ln 2$. [3]

(ii) The region shaded in the diagram is enclosed by the curve and the lines $x = 0$, $x = \ln 2$ and $y = 0$.
Use integration to show that the area of the shaded region is $\frac{5}{2}$. [4]
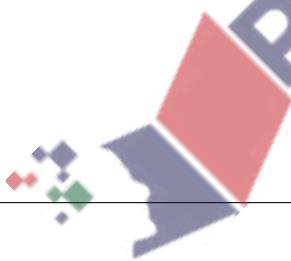
185. 9709_s15_qp_22   Q: 6
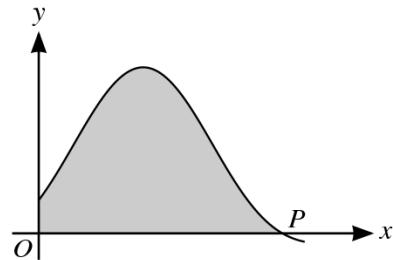
The diagram shows part of the curve with equation

$$y = 4\sin^2 x + 8\sin x + 3$$

and its point of intersection P with the x-axis.

(i) Find the exact x-coordinate of P. [3]

(ii) Show that the equation of the curve can be written

$$y = 5 + 8\sin x - 2\cos 2x,$$

and use integration to find the exact area of the shaded region enclosed by the curve and the axes. [6]

186. 9709_w15_qp_21 Q: 5

(a) Find $\int (\tan^2 x + \sin 2x)\,dx.$ [3]

(b) Find the exact value of $\int_0^1 3e^{1-2x}\,dx.$ [4]
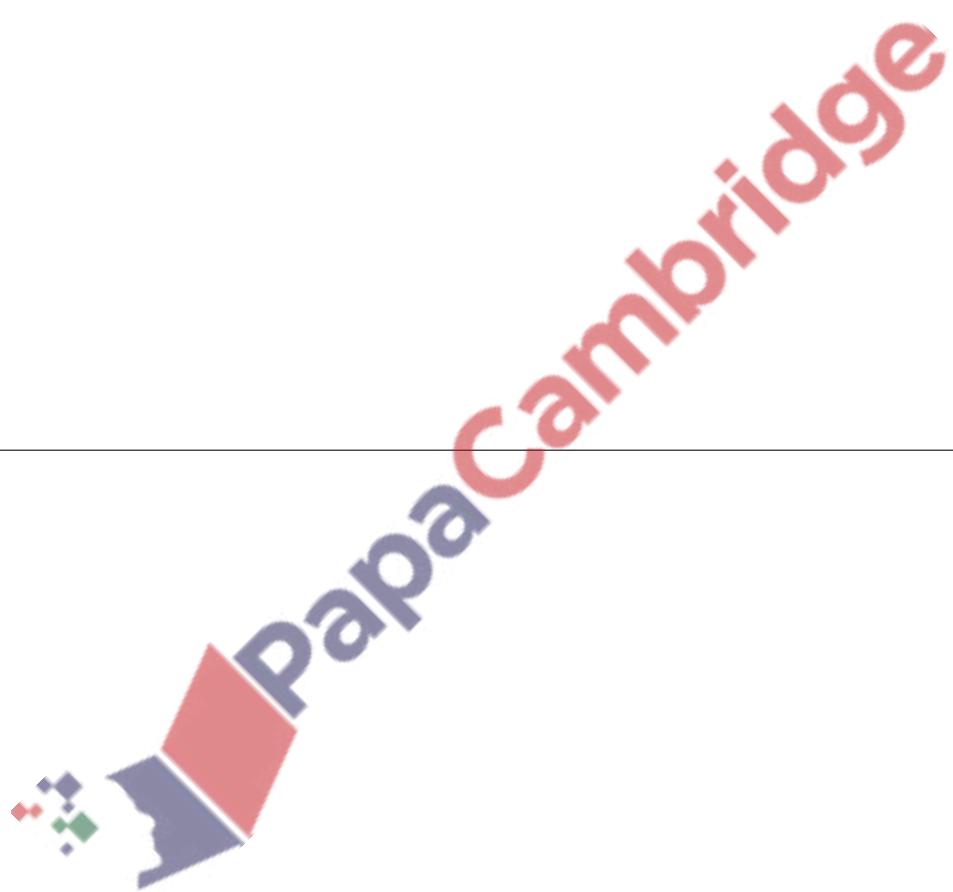
187. 9709_w15_qp_22 Q: 7

(i) Show that the exact value of $\displaystyle\int_0^{\frac{1}{3}\pi} \left(\cos^2 x + \frac{1}{\cos^2 x}\right) \mathrm{d}x$ is $\frac{1}{6}\pi + \frac{9}{8}\sqrt{3}.$ [6]

(ii)
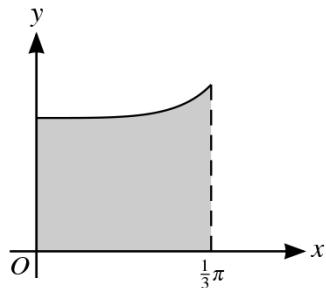
The diagram shows the curve $y = \cos x + \dfrac{1}{\cos x}$ for $0 \leqslant x \leqslant \frac{1}{3}\pi$. The shaded region is bounded by the curve and the lines $x = 0$, $x = \frac{1}{3}\pi$ and $y = 0$. Find the exact volume of the solid obtained when the shaded region is rotated completely about the x-axis. [4]

188. 9709_w15_qp_23 Q: 1

Find the exact value of $\displaystyle\int_{-1}^{35} \frac{3}{2x+5}\,\mathrm{d}x$, giving the answer in the form $\ln k$.   [5]