

Cambridge International AS & A Level

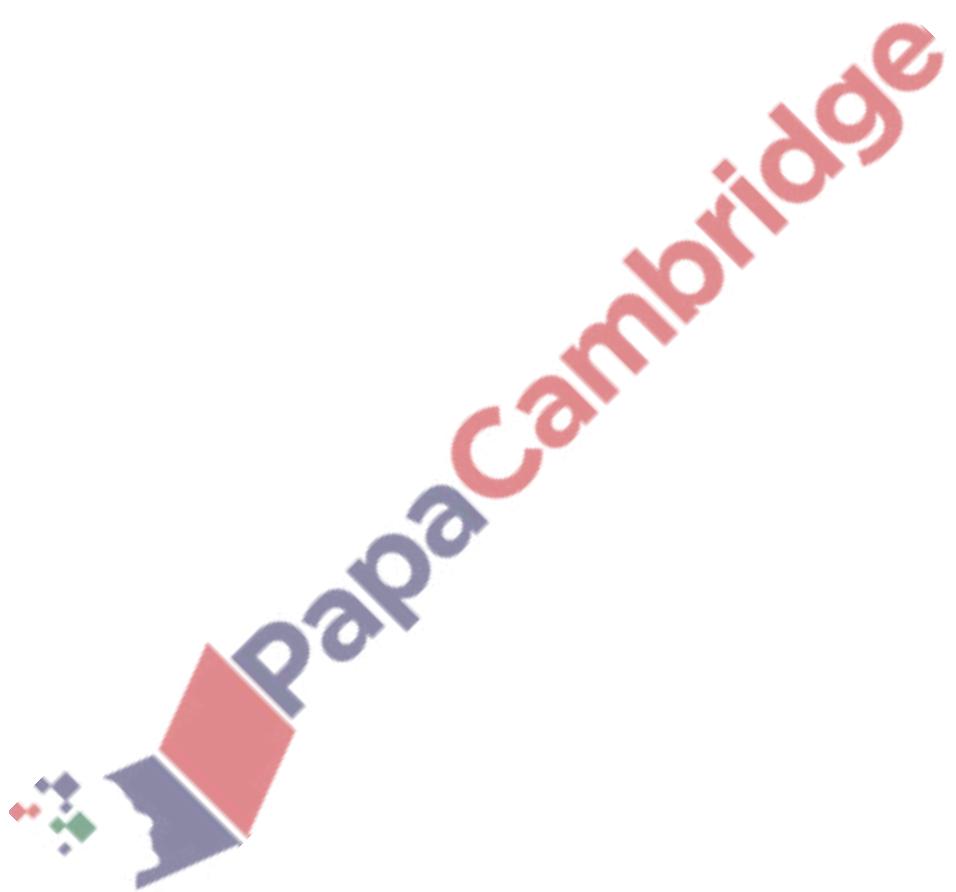
MATHEMATICS (9709) P2

TOPIC WISE QUESTIONS + ANSWERS | COMPLETE SYLLABUS



Chapter 5

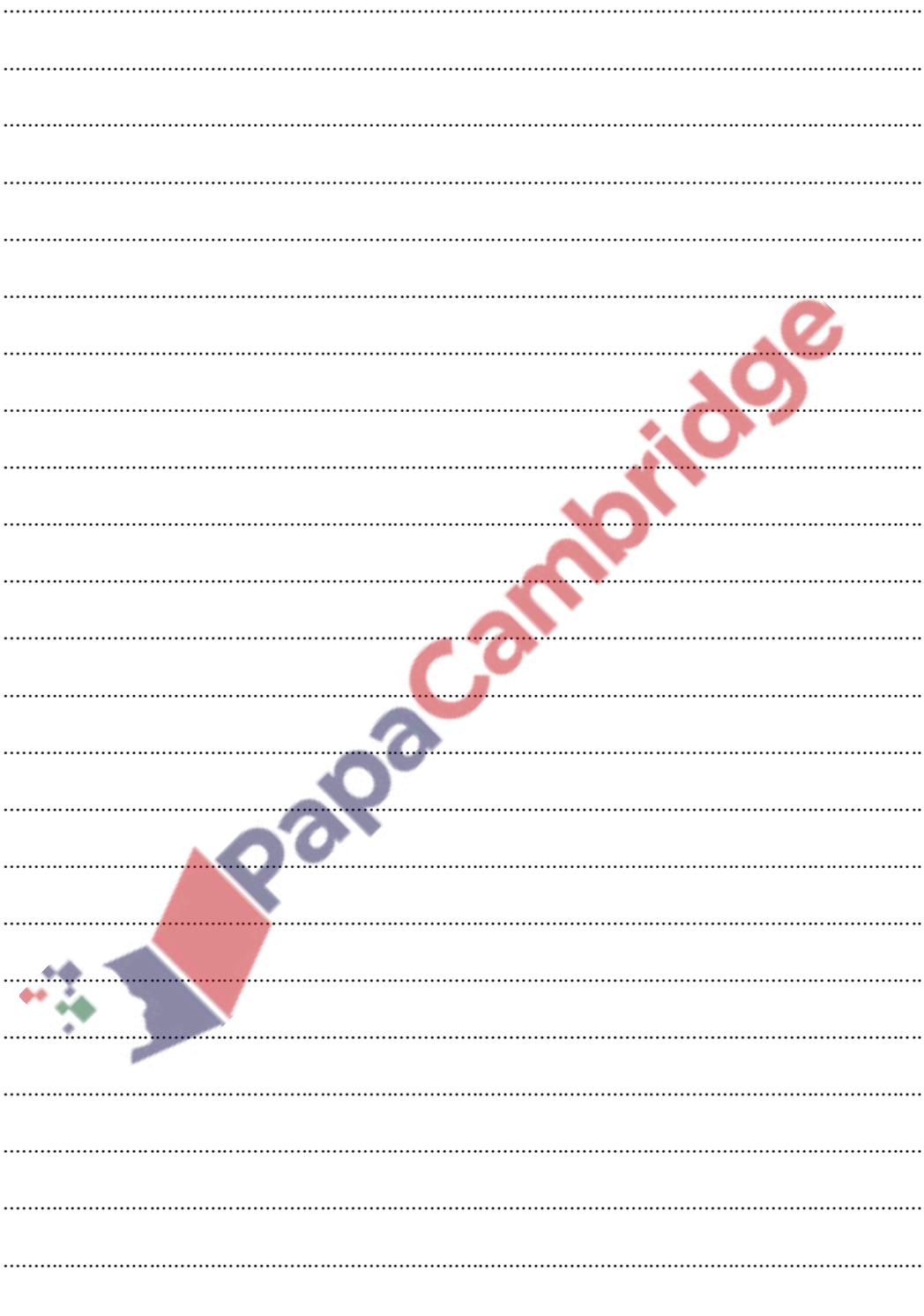
Integration



146. 9709_s20_qp_21_Q: 6

(a) Prove that

$$\sin 2\theta(\operatorname{cosec} \theta - \sec \theta) \equiv \sqrt{8} \cos\left(\theta + \frac{1}{4}\pi\right). \quad [5]$$



(b) Solve the equation

$$\sin 2\theta(\operatorname{cosec} \theta - \sec \theta) = 1$$

for $0 < \theta < \frac{1}{2}\pi$. Give the answer correct to 3 significant figures.

[2]

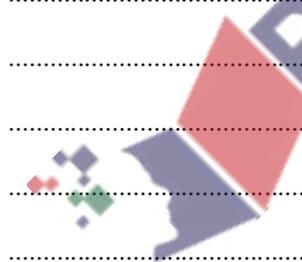
- (c) Find $\int \sin x (\operatorname{cosec} \frac{1}{2}x - \sec \frac{1}{2}x) dx$.

[3]

147. 9709_s20_qp_21 Q: 7

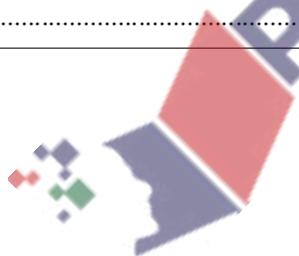
- (a) Find the quotient when $9x^3 - 6x^2 - 20x + 1$ is divided by $(3x + 2)$, and show that the remainder is 9. [3]

- (b) Hence find $\int_1^6 \frac{9x^3 - 6x^2 - 20x + 1}{3x + 2} dx$, giving the answer in the form $a + \ln b$ where a and b are integers. [5]



A large, stylized graphic of the word "Pain" in blue and red, with a small cluster of colorful dots below it.

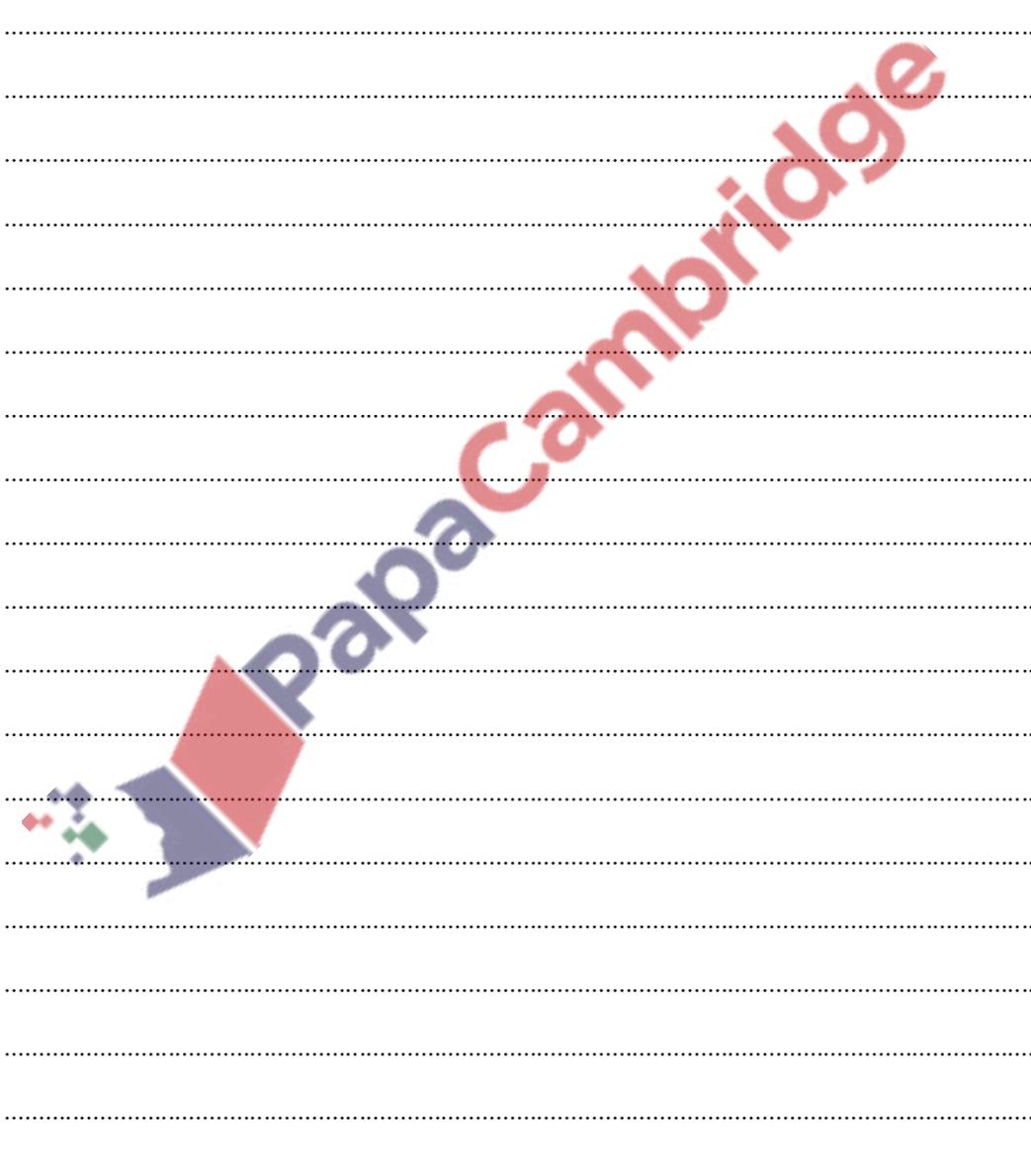
- (c) Find the exact root of the equation $9e^{9y} - 6e^{6y} - 20e^{3y} - 8 = 0$. [4]



148. 9709 - s20 - qp - 22 Q: 7

It is given that $\int_0^a \left(\frac{4}{2x+1} + 8x \right) dx = 10$, where a is a positive constant.

- (a) Show that $a = \sqrt{2.5 - 0.5 \ln(2a + 1)}$. [4]



- (b) Using the equation in part (a), show by calculation that $1 < \alpha < 2$. [2]

- (c) Use an iterative formula, based on the equation in part (a), to find the value of a correct to 4 significant figures. Give the result of each iteration to 6 significant figures. [3]

A red pencil is shown writing the word "Papacamb" diagonally across a sheet of white paper with horizontal ruling lines. The pencil has a blue eraser at the top and a small decorative pattern of colored squares (red, green, blue) near the bottom. The background is plain white.

149. 9709_s20_qp_22 Q: 8

- (a) Show that $3 \sin 2\theta \cot \theta \equiv 6 \cos^2 \theta$. [2]

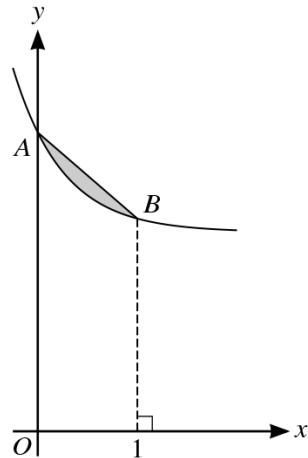
- (b) Solve the equation $3 \sin 2\theta \cot \theta = 5$ for $0 < \theta < \pi$. [3]

The image features a large, stylized graphic element in the upper left corner. It consists of a red trapezoid pointing upwards and a blue trapezoid pointing downwards, which are partially overlapping. Below this, there is a small cluster of small, colorful dots in shades of red, green, and blue.

(c) Find the exact value of $\int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} 3 \sin x \cot \frac{1}{2}x \, dx$. [5]

A large, semi-transparent watermark is positioned diagonally across the page. The watermark features the text "PapaCambridge" in a bold, sans-serif font. The letters are colored in a gradient: "Papa" is in light blue, and "Cambridge" is in red. Below the text is a graphic element consisting of a blue arrow pointing to the right, with a red shape attached to its left side, resembling a stylized 'A' or a sparkler.

150. 9709_w20_qp_21 Q: 3



The diagram shows the curve $y = 2 + e^{-2x}$. The curve crosses the y -axis at the point A , and the point B on the curve has x -coordinate 1. The shaded region is bounded by the curve and the line segment AB .

Find the exact area of the shaded region.

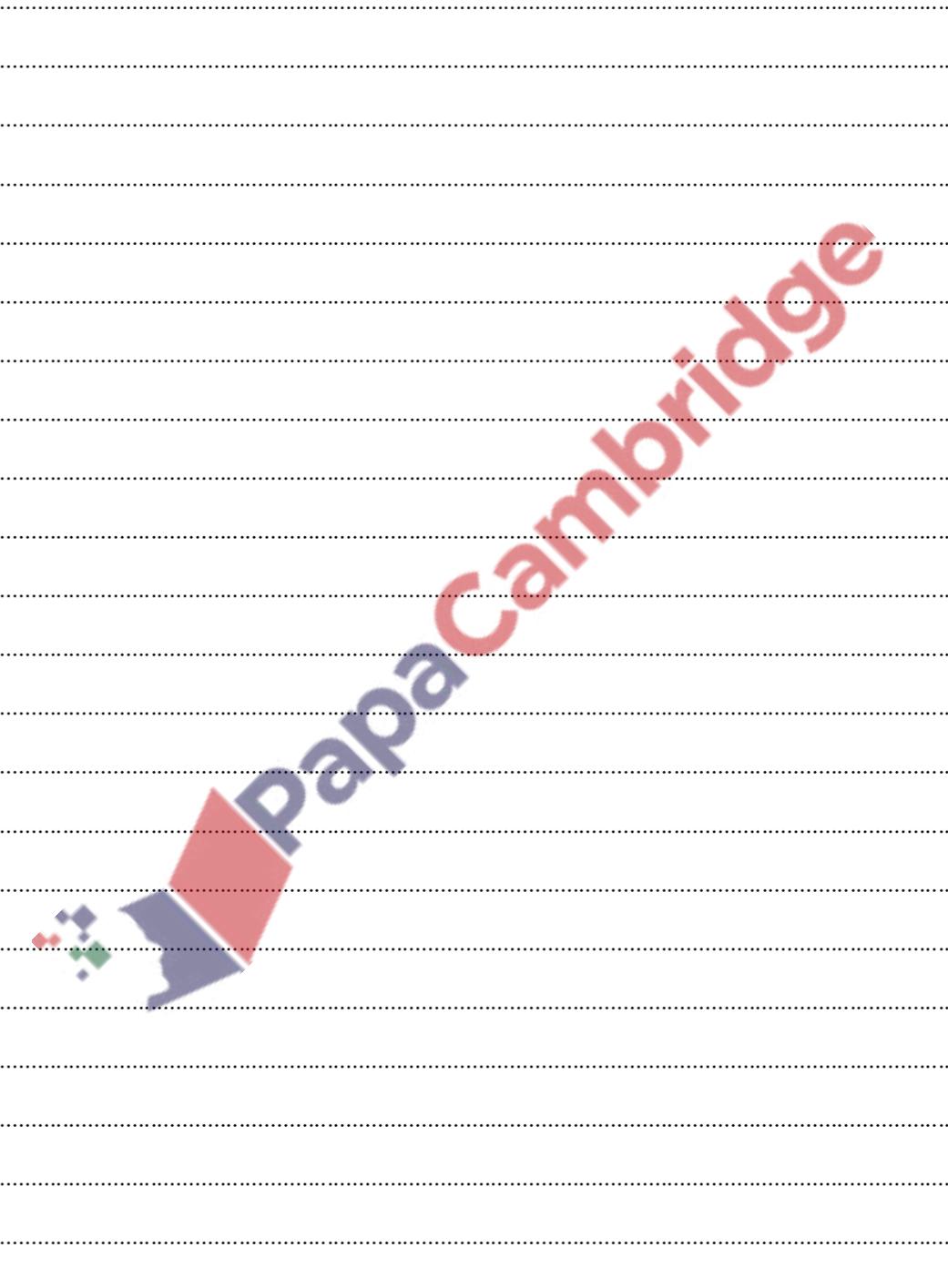
[5]



151. 9709_w20_qp_21 Q: 8

A curve has equation $y = f(x)$ where $f(x) = \frac{4x^3 + 8x - 4}{2x - 1}$.

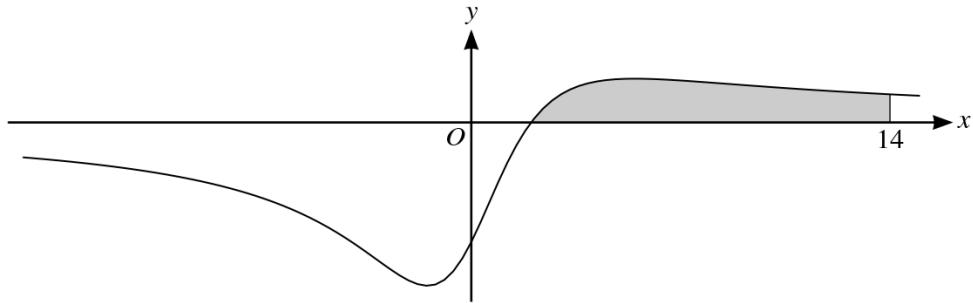
- (a) Find an expression for $\frac{dy}{dx}$ and hence find the coordinates of each of the stationary points of the curve $y = f(x)$. [5]



- (b) Divide $4x^3 + 8x - 4$ by $(2x - 1)$, and hence find $\int f(x) dx$. [5]

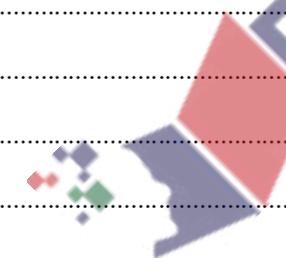
A large, semi-transparent watermark is positioned diagonally across the page. The watermark features the text "PapaCambridge" in a bold, sans-serif font, with "Papa" in grey and "Cambridge" in red. Below the text is a graphic element consisting of a blue pen tip pointing upwards and to the right, with a trail of small, colorful dots (red, green, blue) extending from its tip.

152. 9709_w20_qp_22 Q: 4



The diagram shows the curve with equation $y = \frac{x-2}{x^2+8}$. The shaded region is bounded by the curve and the lines $x = 14$ and $y = 0$.

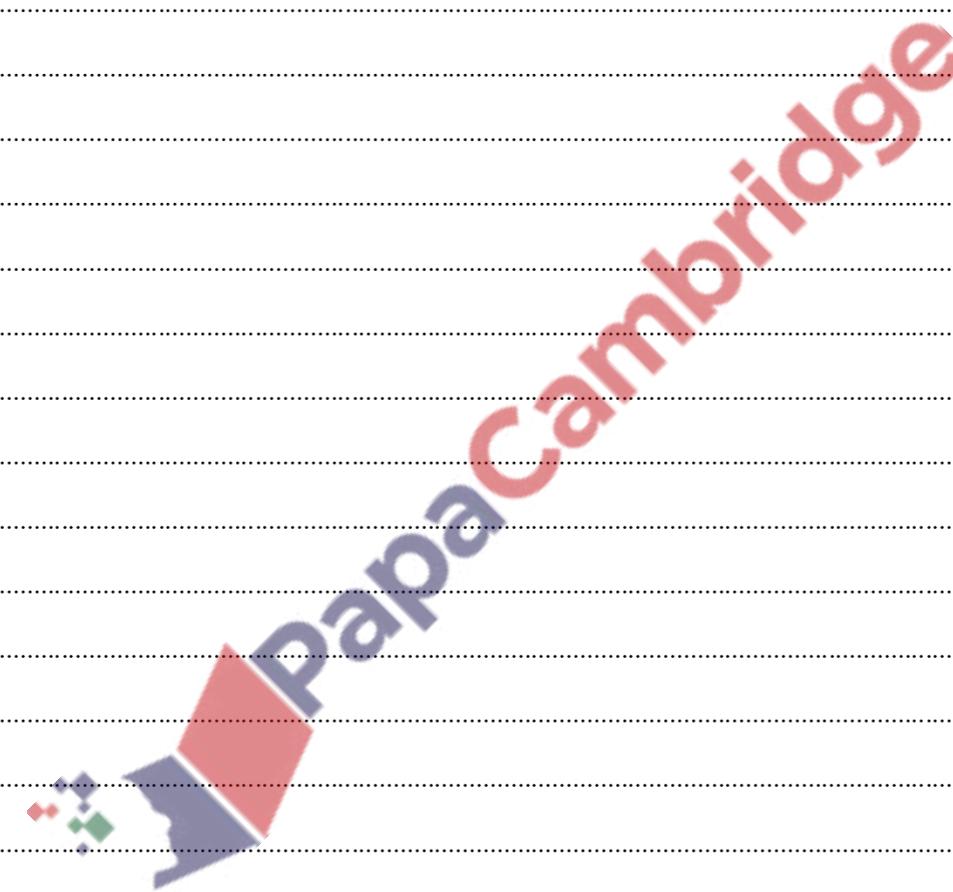
- (a) Find $\frac{dy}{dx}$ and hence determine the exact x -coordinates of the stationary points. [4]



- (b) Use the trapezium rule with three intervals to find an approximation to the area of the shaded region. Give the answer correct to 2 significant figures. [3]

153. 9709_w20_qp_22 Q: 6

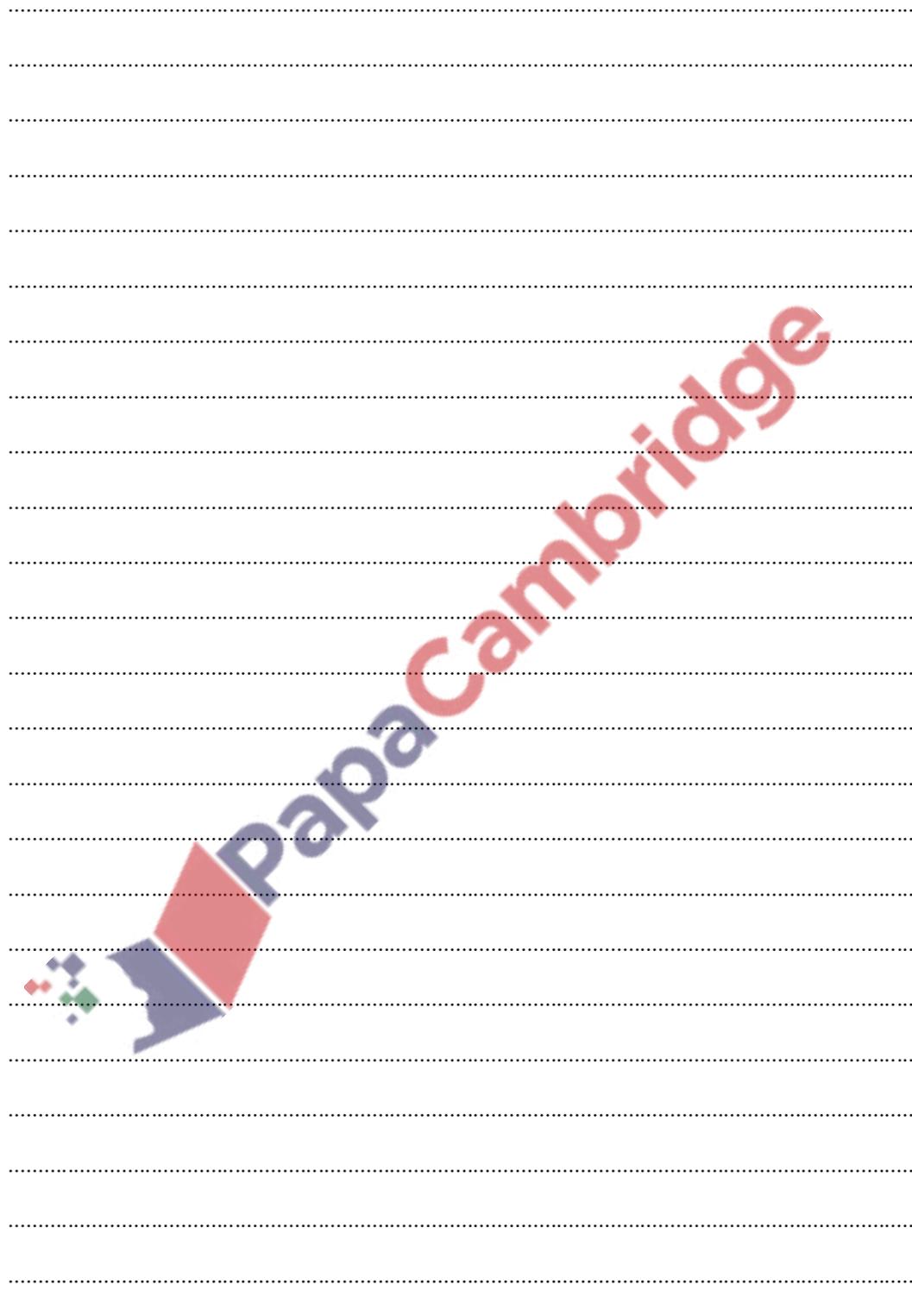
(a) Find $\int \left(\frac{8}{4x+1} + \frac{8}{\cos^2(4x+1)} \right) dx$. [4]



- (b)** It is given that $\int_0^{\frac{1}{2}\pi} (3 + 4 \cos^2 \frac{1}{2}x + k \sin 2x) dx = 10$.

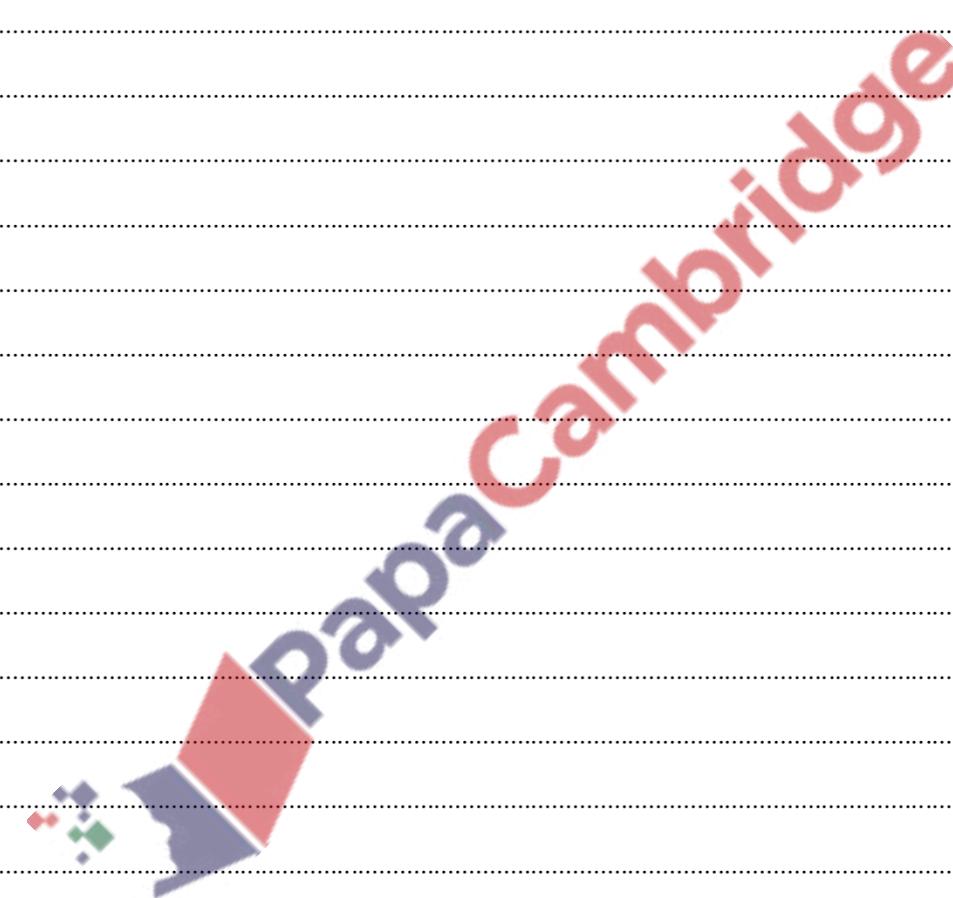
Find the exact value of the constant k .

[6]



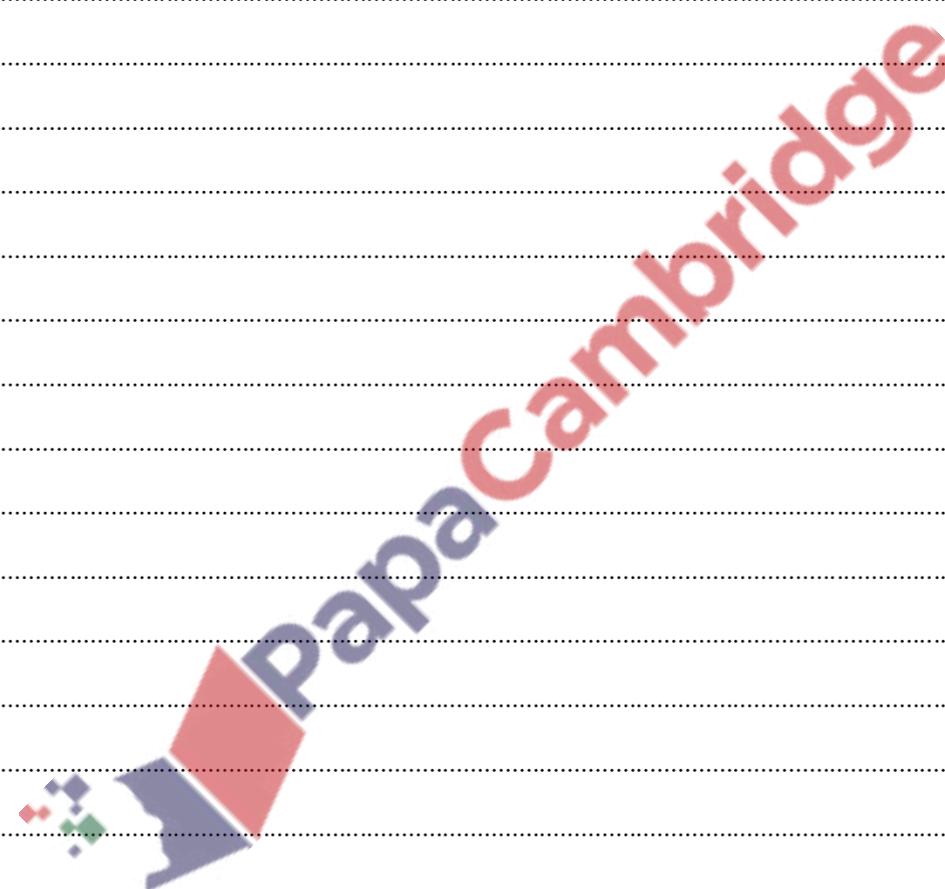
154. 9709_m19_qp_22 Q: 6

(a) Show that $\int_1^4 \left(\frac{2}{x} + \frac{2}{2x+1} \right) dx = \ln 48.$ [5]



(b) Find $\int \sin 2x (\cot x + 2 \operatorname{cosec} x) dx$.

[6]



155. 9709_s19_qp_21 Q: 4

- (a)** Find $\int \tan^2 3x \, dx$. [3]

bridge

- (b)** Find the exact value of $\int_0^1 \frac{e^{3x} + 4}{e^x} dx$. Show all necessary working. [4]

The logo for Papacat features a large, stylized red and grey cat head at the top, with the word "Papacat" written diagonally across it in a matching font. Below the head is a blue and red geometric shape resembling a stylized cat's body or a ribbon. To the left of this shape is a small graphic element consisting of a purple triangle above a green hexagon, with a red diamond to its left.

156. 9709_s19_qp_22 Q: 4

- (a) Find the exact value of $\int_0^{\frac{1}{2}\pi} (4 \sin 2x + 2 \cos^2 x) dx$. Show all necessary working. [5]

- (b) Use the trapezium rule with two intervals to find an approximation to $\int_2^8 \sqrt{(\ln(1+x))} dx$. [3]

A decorative graphic element located in the top left corner of the page. It consists of a large, stylized, abstract shape composed of overlapping red and blue geometric forms. To the left of this main shape, there is a cluster of smaller, semi-transparent colored dots in shades of red, green, and blue.

157. 9709_s19_qp_22 Q: 5

- (i) Find the quotient and remainder when $2x^3 + x^2 - 8x$ is divided by $(2x + 1)$. [3]



- (ii) Hence find the exact value of $\int_0^3 \frac{2x^3 + x^2 - 8x}{2x + 1} dx$, giving the answer in the form $\ln(ke^a)$ where k and a are constants. [5]

A large, semi-transparent watermark is positioned diagonally across the page. The watermark features the text "PapaCambridge" in a bold, red, sans-serif font. To the left of the text is a stylized graphic of a pencil. The pencil has a blue eraser end and a red body. At the top of the pencil, there is a small cluster of colorful dots in shades of red, green, and blue, resembling a flower or a burst of energy.

158. 9709_w19_qp_21 Q: 2

Find the exact value of $\int_1^2 (2e^{2x} - 1)^2 dx$. Show all necessary working. [5]

159. 9709_w19_qp_22 Q: 6

- (a) Show that $\int_2^{18} \frac{3}{2x} dx = \ln 27$. [4]

- (b) Find the exact value of $\int_0^{\frac{1}{6}\pi} 4 \sin^2\left(\frac{3}{2}x\right) dx$. Show all necessary working. [5]

The logo for PapaCasa features the brand name in a large, stylized font where the letters 'P', 'a', 'b', 'a', 'C', and 'a' are stacked vertically. The 'P' is blue, the 'a's are red, the 'b' is grey, and the 'C' and 'a' are red. Below the main text is a graphic element consisting of a blue triangle pointing right, a red triangle pointing right, and a cluster of small colored dots (red, green, blue) at the base.

160. 9709_m18_qp_22 Q: 3

- (i) Use the trapezium rule with four intervals to find an approximation to

$$\int_0^8 \ln(x+2) \, dx,$$

giving your answer correct to 3 significant figures.

[3]

(ii) Hence find an approximation to

$$\int_0^8 3 \ln(x^2 + 4x + 4) dx.$$

[2]

161. 9709_m18_qp_22 Q: 6

- (i) Show that $\operatorname{cosec} 2x + \cot 2x \equiv \cot x$. [4]



(ii) Hence find the exact value of $\cot \frac{1}{12}\pi$.

[2]

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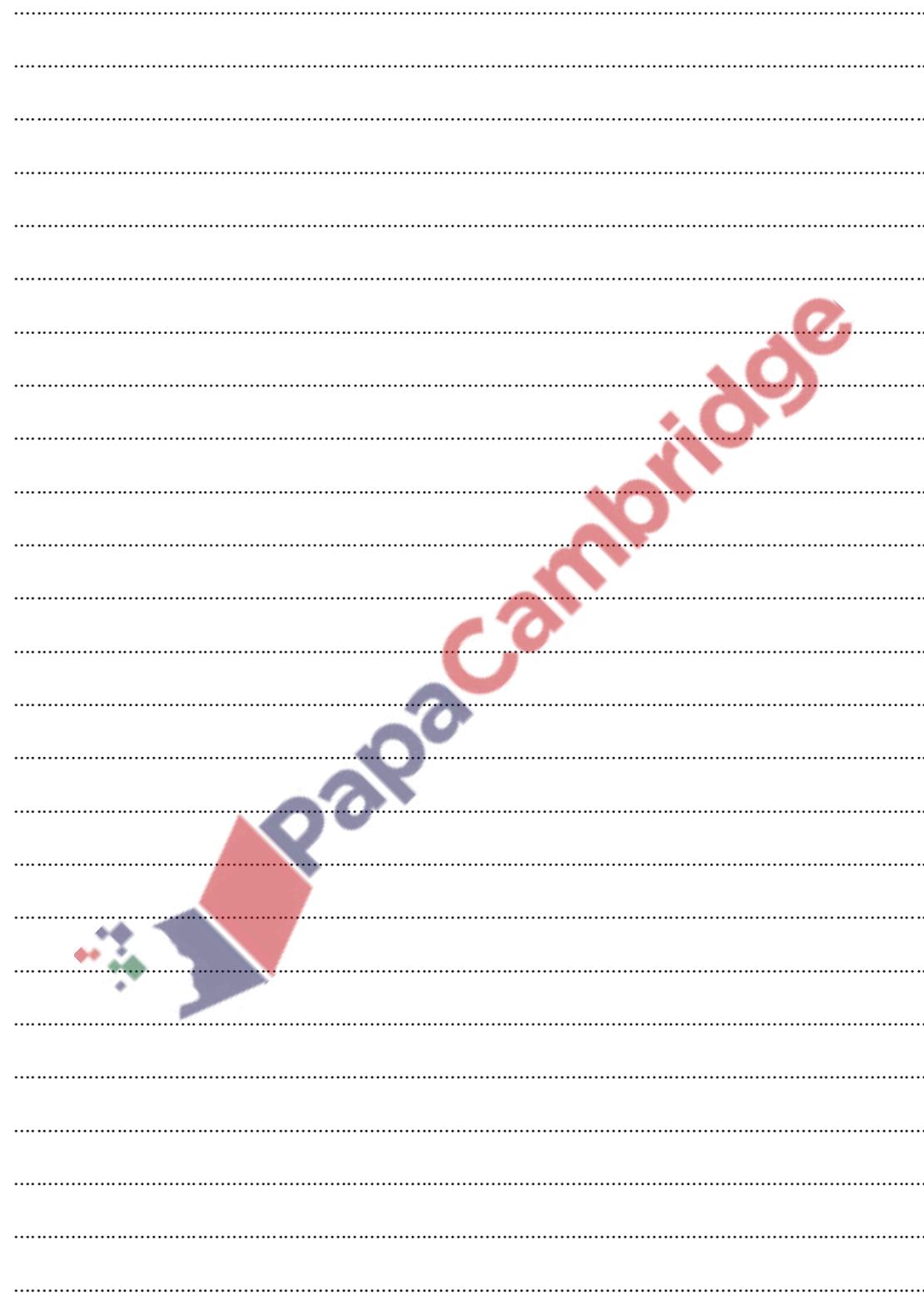
(iii) Find $\int \sin 2x(\operatorname{cosec} 4x + \cot 4x) dx$.

[3]

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162. 9709_s18_qp_21_Q: 3

Without using a calculator, find the exact value of $\int_0^2 4e^{-x}(e^{3x} + 1) dx$. [5]



163. 9709_s18_qp_21 Q: 7

- (i) Express $5 \cos \theta - 2 \sin \theta$ in the form $R \cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Give the value of α correct to 4 decimal places. [3]

- (ii)** Using your answer from part **(i)**, solve the equation

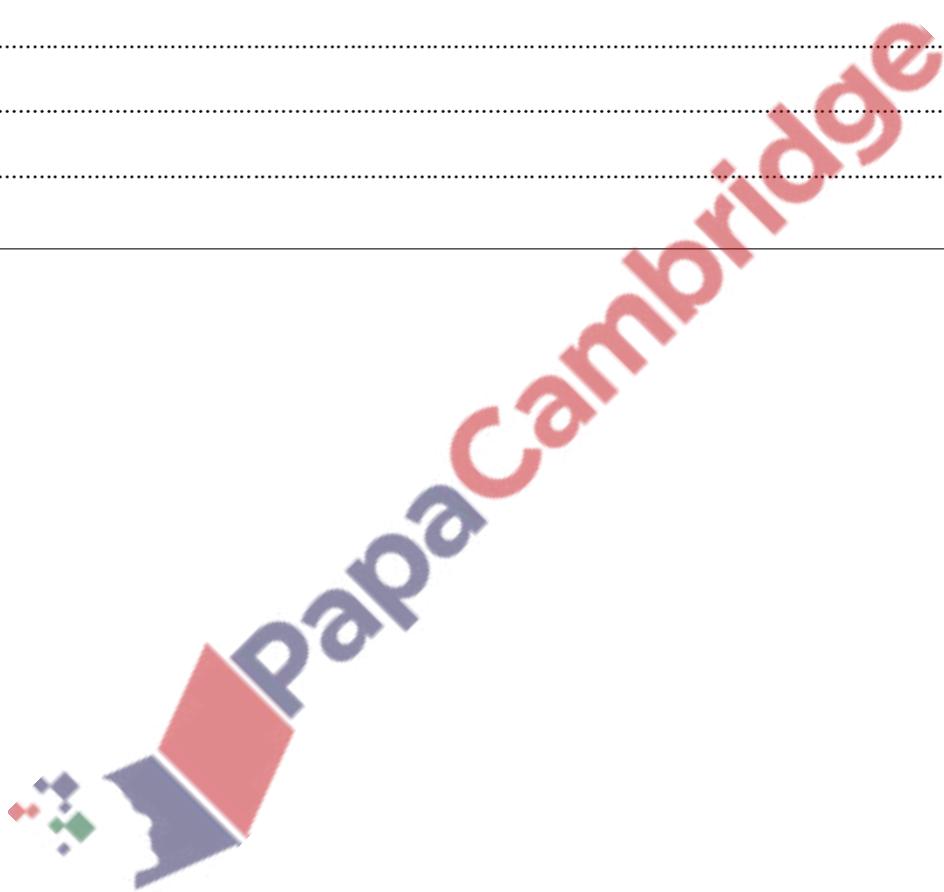
$$5 \cot \theta - 4 \operatorname{cosec} \theta = 2$$

for $0 < \theta < 2\pi$.

[5]

A decorative graphic element located at the top left of the page. It features a stylized map of the state of Maryland in white against a light gray background. Overlaid on the map are several geometric shapes: a red triangle pointing upwards, a green diamond shape, and a blue square. To the left of the map, there is a cluster of smaller, overlapping shapes in red, green, and blue.

(iii) Find $\int \frac{1}{(5 \cos \frac{1}{2}x - 2 \sin \frac{1}{2}x)^2} dx.$ [3]



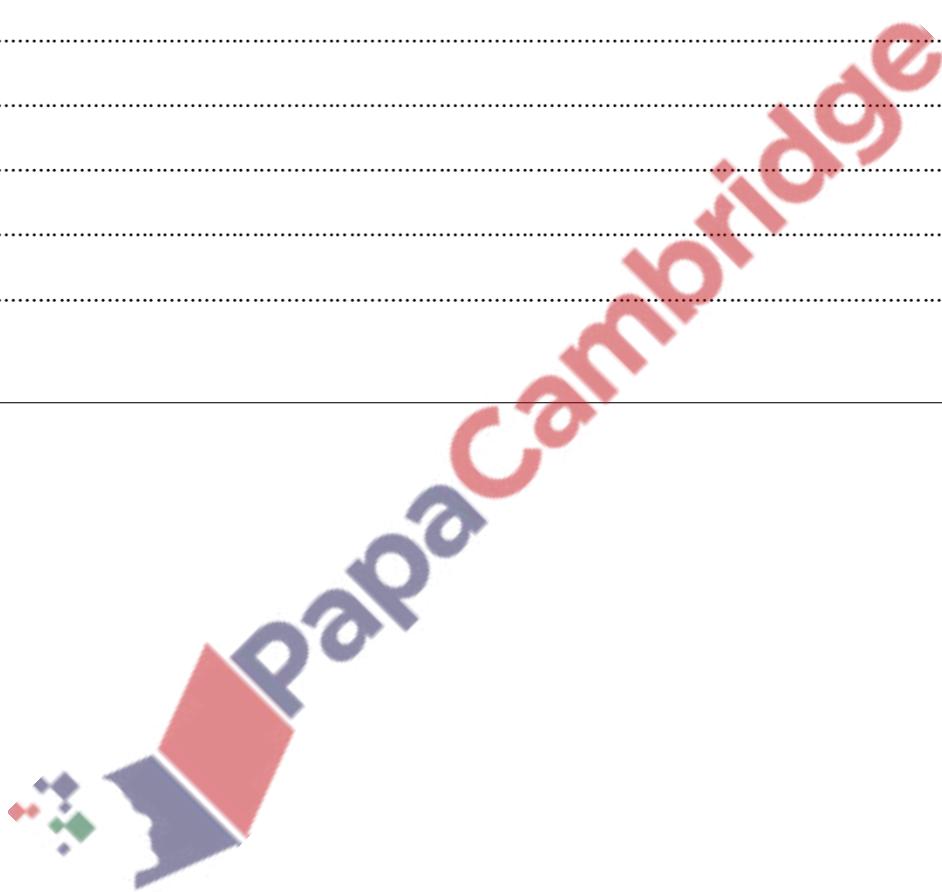
164. 9709_s18_qp_22 Q: 7

- (i) Show that $2 \operatorname{cosec}^2 2x(1 - \cos 2x) \equiv \sec^2 x$. [3]

- (ii) Solve the equation $2 \operatorname{cosec}^2 2x(1 - \cos 2x) = \tan x + 21$ for $0 < x < \pi$, giving your answers correct to 3 significant figures. [4]

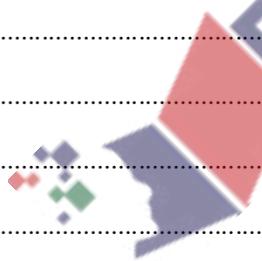


(iii) Find $\int [2 \operatorname{cosec}^2(4y + 2) - 2 \operatorname{cosec}^2(4y + 2) \cos(4y + 2)] dy$. [3]

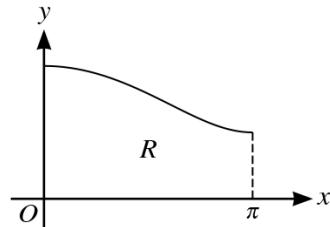


165. 9709_w18_qp_21 Q: 2

Show that $\int_1^7 \frac{6}{2x+1} dx = \ln 125.$ [5]

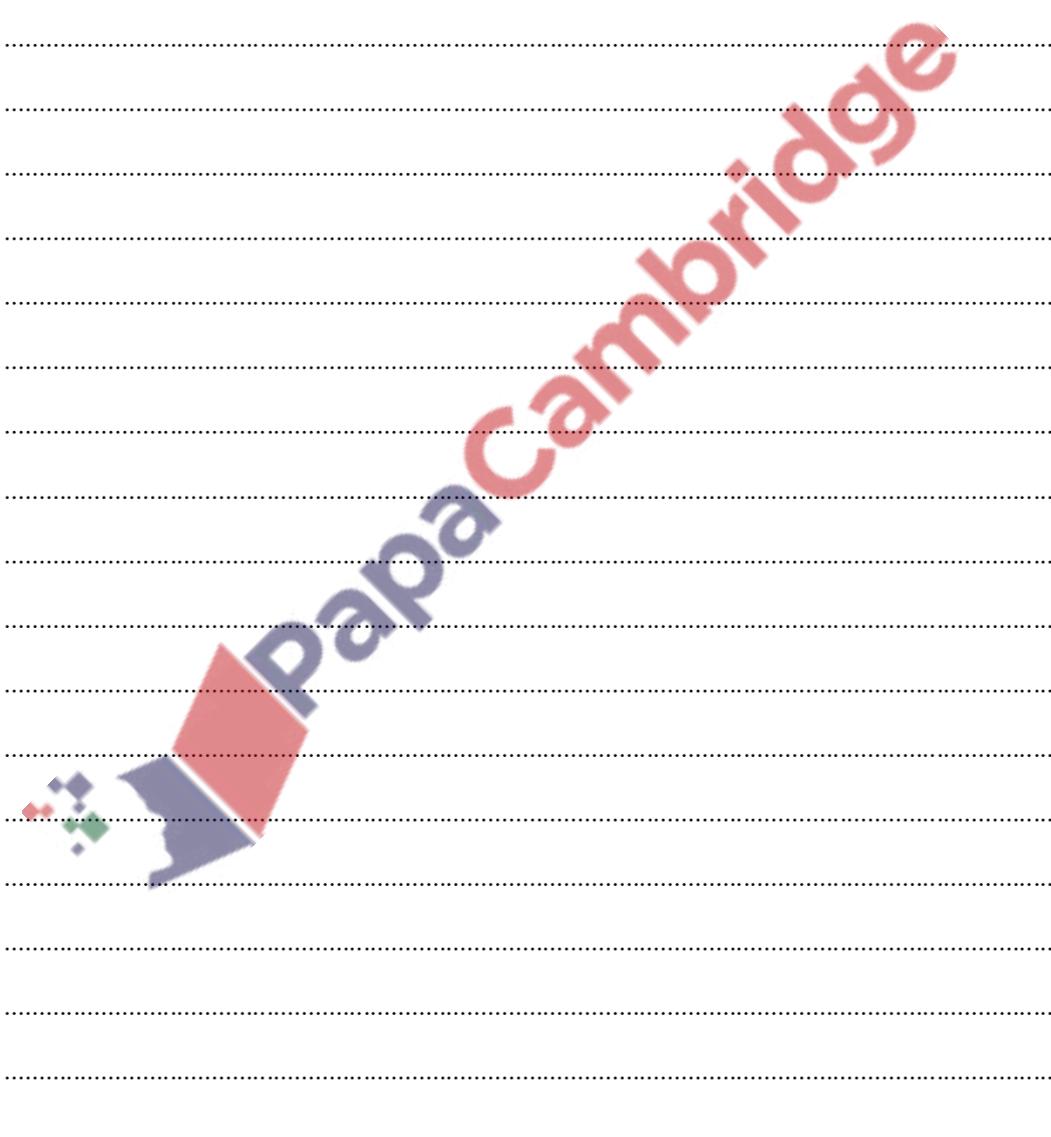


166. 9709_w18_qp_21 Q: 6



The diagram shows the curve with equation $y = \sqrt{(1 + 3\cos^2(\frac{1}{2}x))}$ for $0 \leq x \leq \pi$. The region R is bounded by the curve, the axes and the line $x = \pi$.

- (i) Use the trapezium rule with two intervals to find an approximation to the area of R , giving your answer correct to 3 significant figures. [3]

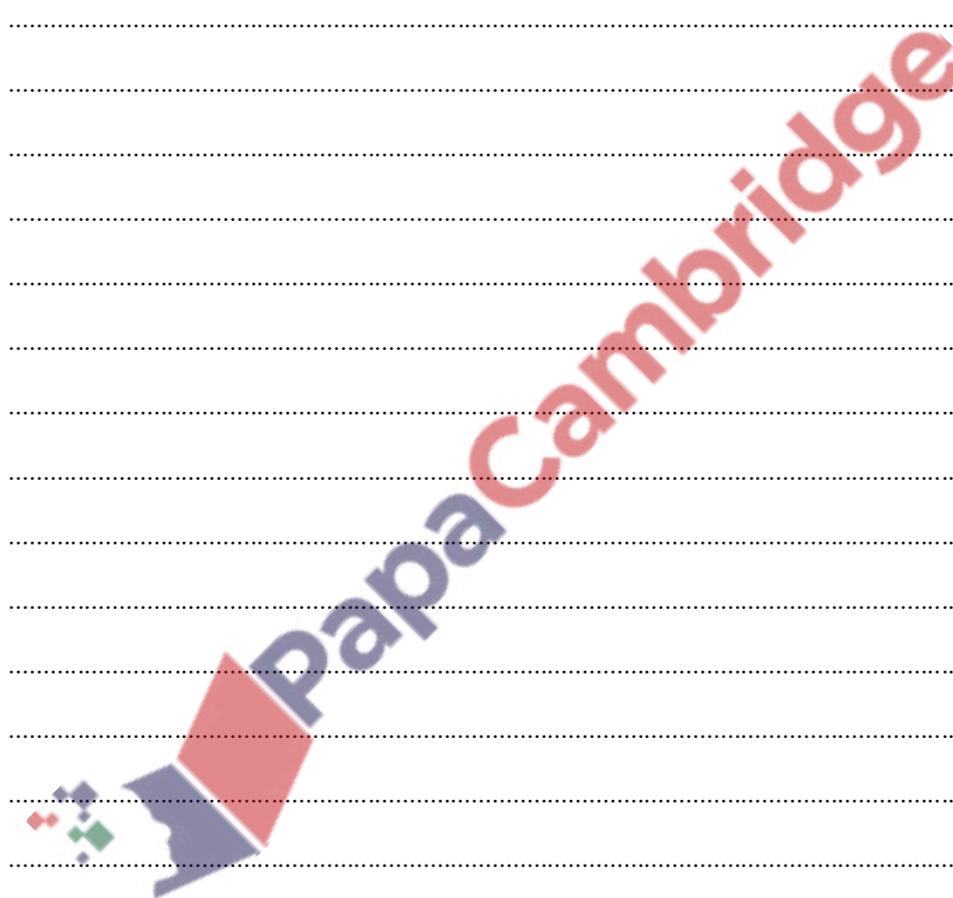


- (ii) The region R is rotated completely about the x -axis. Without using a calculator, find the exact volume of the solid produced. [5]

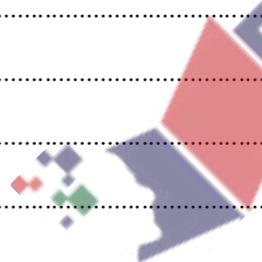
A large, semi-transparent watermark is positioned diagonally across the page. The watermark features the text "PapaCambridge" in a bold, sans-serif font. The letters are colored in a gradient: blue for "Papa", red for "Cam", and orange for "bridge". Below the text is a stylized graphic of a pen or pencil. The pen has a blue barrel, a pink eraser, and a dark blue cap. At the base of the pen, there is a cluster of small, colorful dots in green, red, and blue.

167. 9709_w18_qp_22 Q: 6

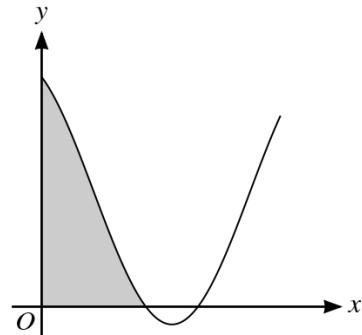
- (a) Show that $\int_1^6 \frac{12}{3x+2} dx = \ln 256$. [5]



- (b) Find the exact value of $\int_0^{\frac{1}{6}\pi} (8 \sin^2 x + \tan^2 2x) dx$, showing all necessary working. [6]



168. 9709_m17_qp_22 Q: 7



The diagram shows part of the curve

$$y = 2 \cos 2x \cos(2x + \frac{1}{6}\pi).$$

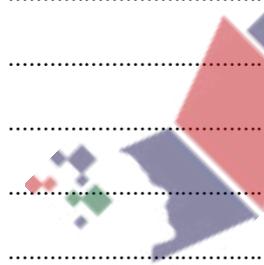
The shaded region is bounded by the curve and the two axes.

- (i) Show that $2 \cos 2x \cos(2x + \frac{1}{6}\pi)$ can be expressed in the form

$$k_1(1 + \cos 4x) + k_2 \sin 4x,$$

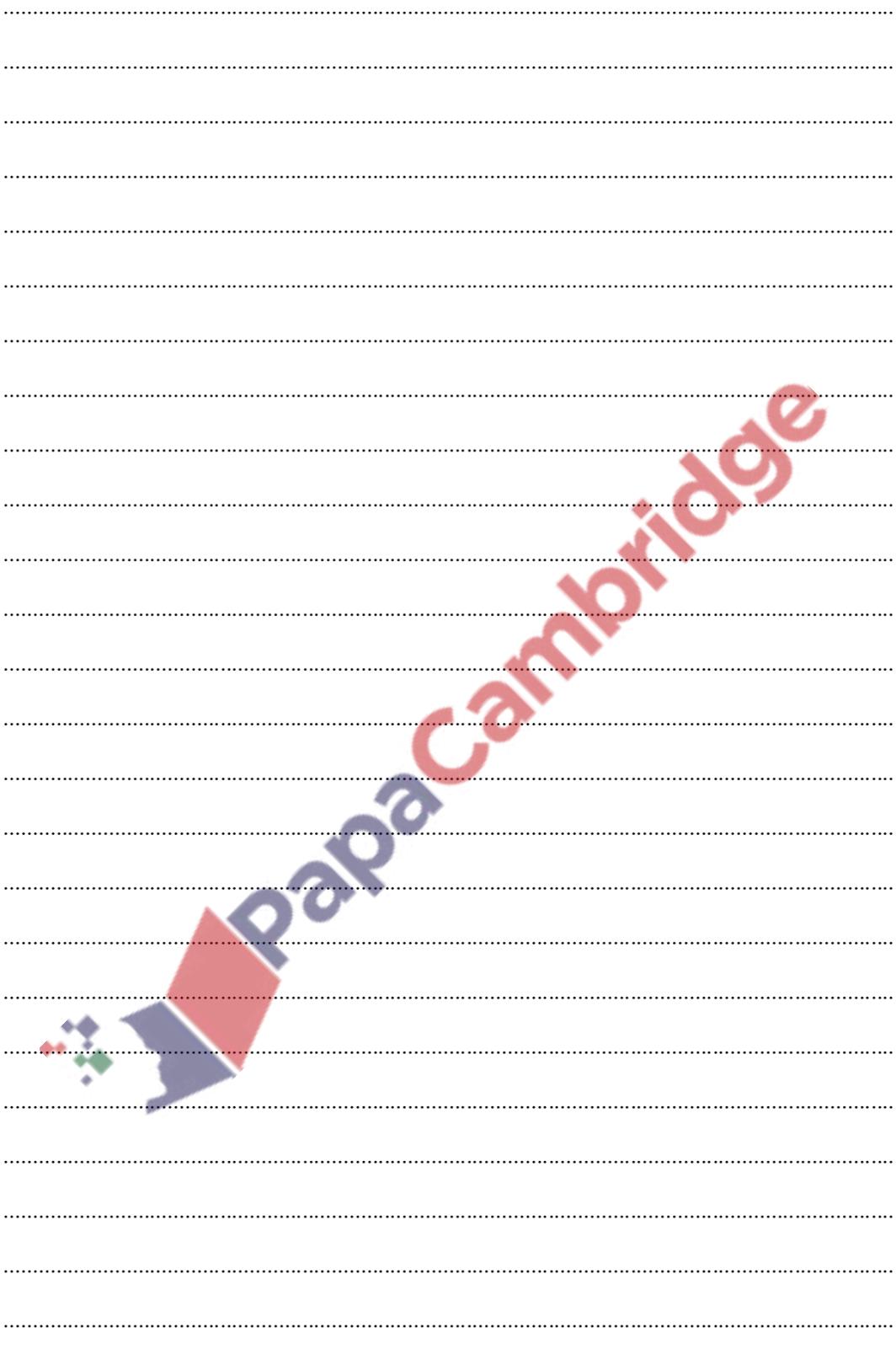
where the values of the constants k_1 and k_2 are to be determined.

[5]



(ii) Find the exact area of the shaded region.

[5]

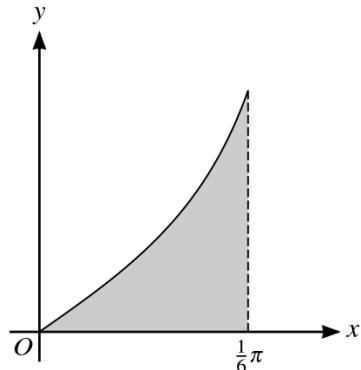


169. 9709 - s17 - qp - 21 Q: 3

Given that $\int_0^a 4e^{\frac{1}{2}x+3} dx = 835$, find the value of the constant a correct to 3 significant figures. [5]

A large, semi-transparent watermark is positioned diagonally across the page. The watermark features the text "PapaCambridge" in a bold, sans-serif font. The letters are colored in a gradient: "Papa" is in blue, and "Cambridge" is in red. Below the text is a stylized graphic of a building or castle tower, rendered in red and blue with small green and pink decorative elements at the base.

170. 9709_s17_qp_21 Q: 6

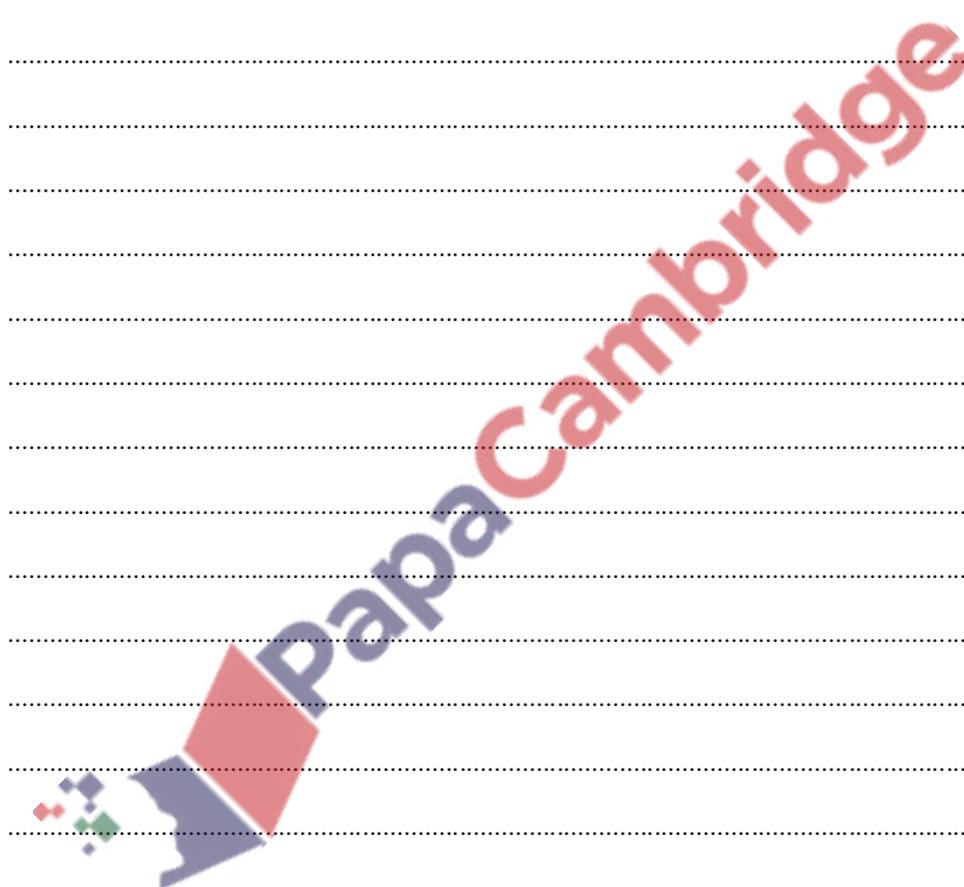


The diagram shows the curve $y = \tan 2x$ for $0 \leq x \leq \frac{1}{6}\pi$. The shaded region is bounded by the curve and the lines $x = \frac{1}{6}\pi$ and $y = 0$.

- (i) Use the trapezium rule with two intervals to find an approximation to the area of the shaded region, giving your answer correct to 3 significant figures. [3]

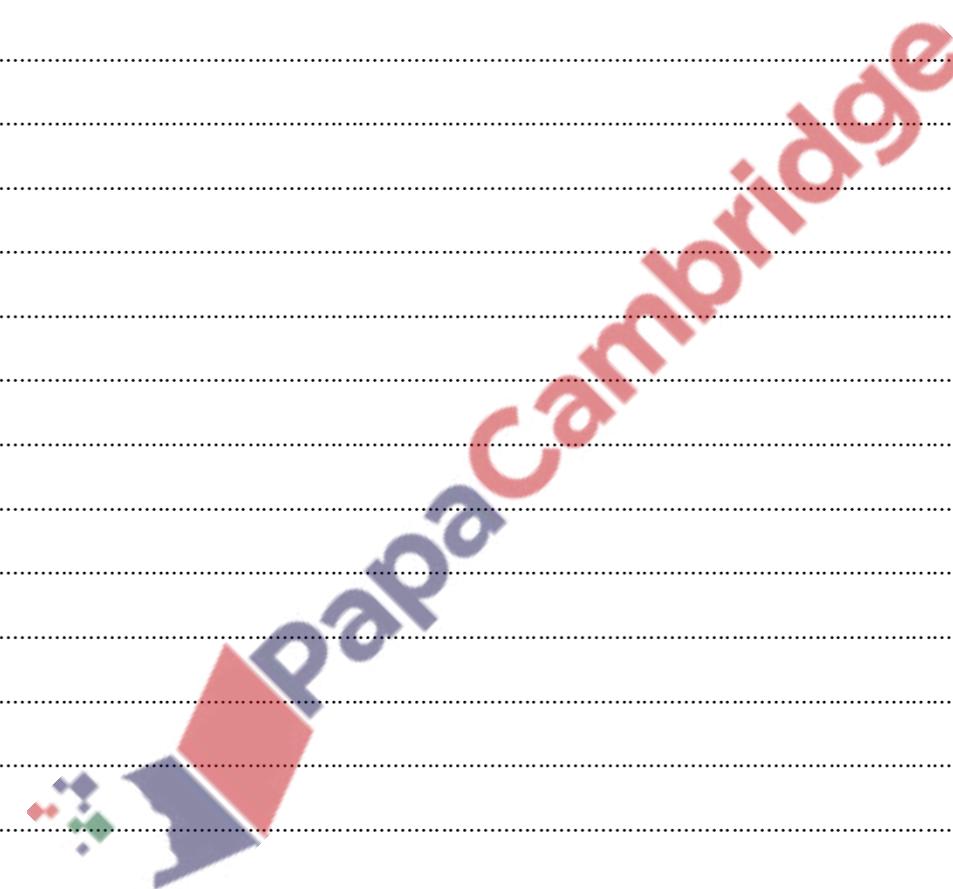


- (ii) Find the exact volume of the solid formed when the shaded region is rotated completely about the x -axis. [4]



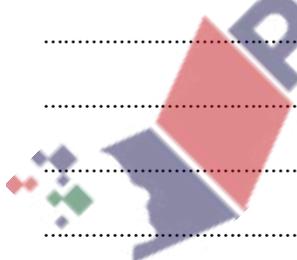
171. 9709 - s17 - qp - 22 Q: 7

- (a) Find $\int (2 \cos \theta - 3)(\cos \theta + 1) d\theta$. [4]



- (b) (i) Find $\int \left(\frac{4}{2x+1} + \frac{1}{2x} \right) dx$. [2]

- (ii) Hence find $\int_1^4 \left(\frac{4}{2x+1} + \frac{1}{2x} \right) dx$, giving your answer in the form $\ln k$. [3]



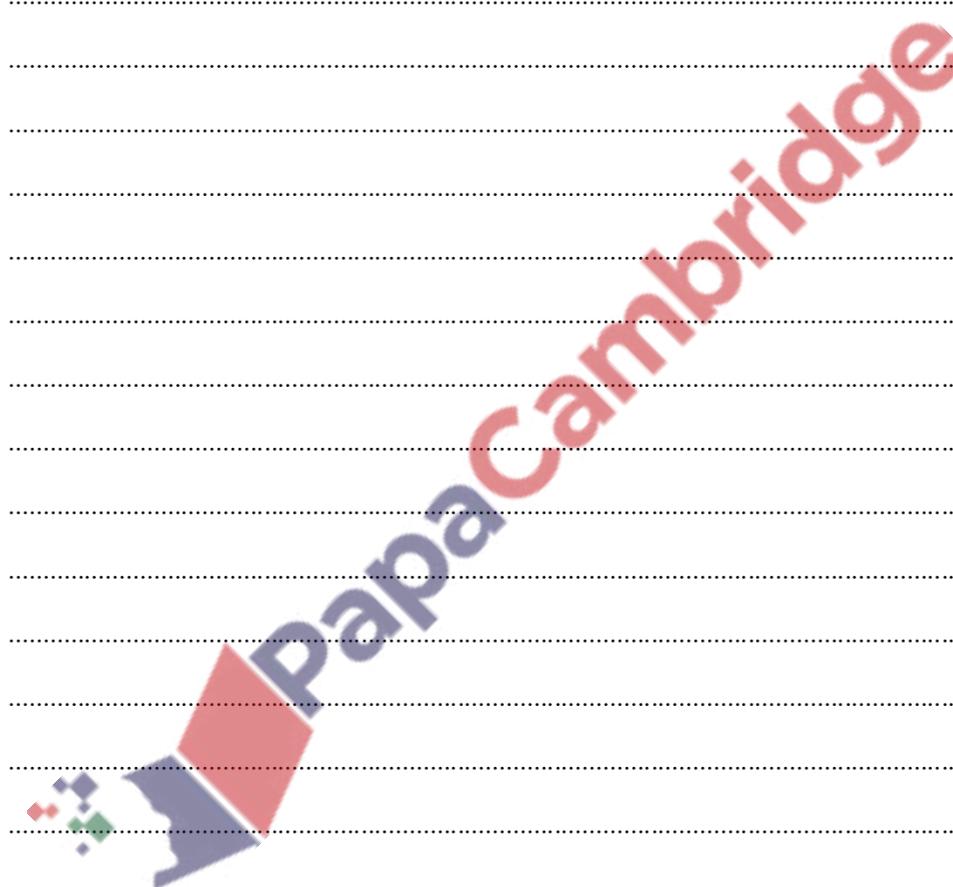
172. 9709_w17_qp_21 Q: 4

(a) Find $\int \frac{4 + \sin^2 \theta}{1 - \sin^2 \theta} d\theta$. [4]

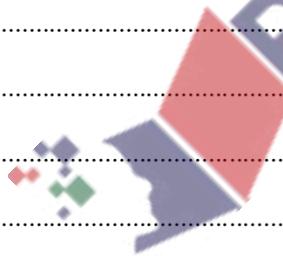
(b) Given that $\int_0^a \frac{2}{3x+1} dx = \ln 16$, find the value of the positive constant a . [4]

173. 9709_w17_qp_22 Q: 6

- (a) Find the exact value of $\int_0^{\frac{1}{4}\pi} \sin x(4 \sin x + 6 \cos x) dx$. [5]



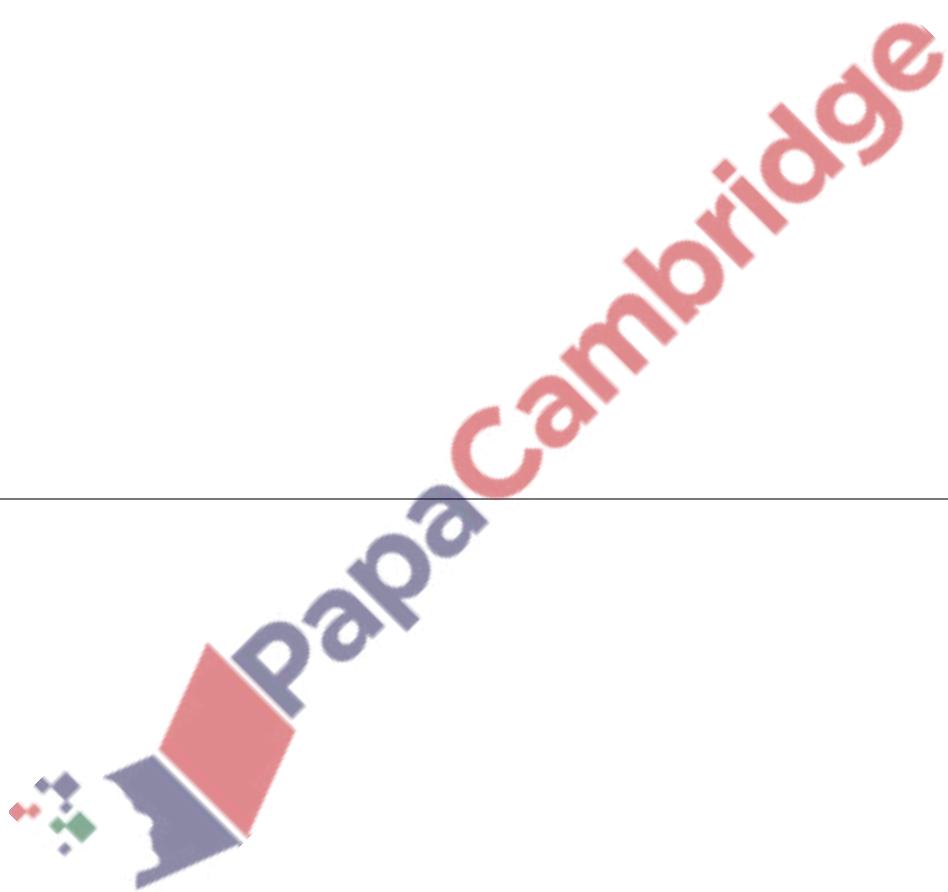
- (b) Given that $\int_0^a \frac{6}{3x+2} dx = \ln 49$, find the value of the positive constant a . [5]



174. 9709_m16_qp_22 Q: 5

Given that $\int_0^a 6e^{2x+1} dx = 65$, find the value of a correct to 3 decimal places.

[5]



175. 9709 _ m16 _ qp _ 22 Q: 8

(i) Show that $\sin 2x \cot x \equiv 2 \cos^2 x$. [2]

(ii) Using the identity in part (i),

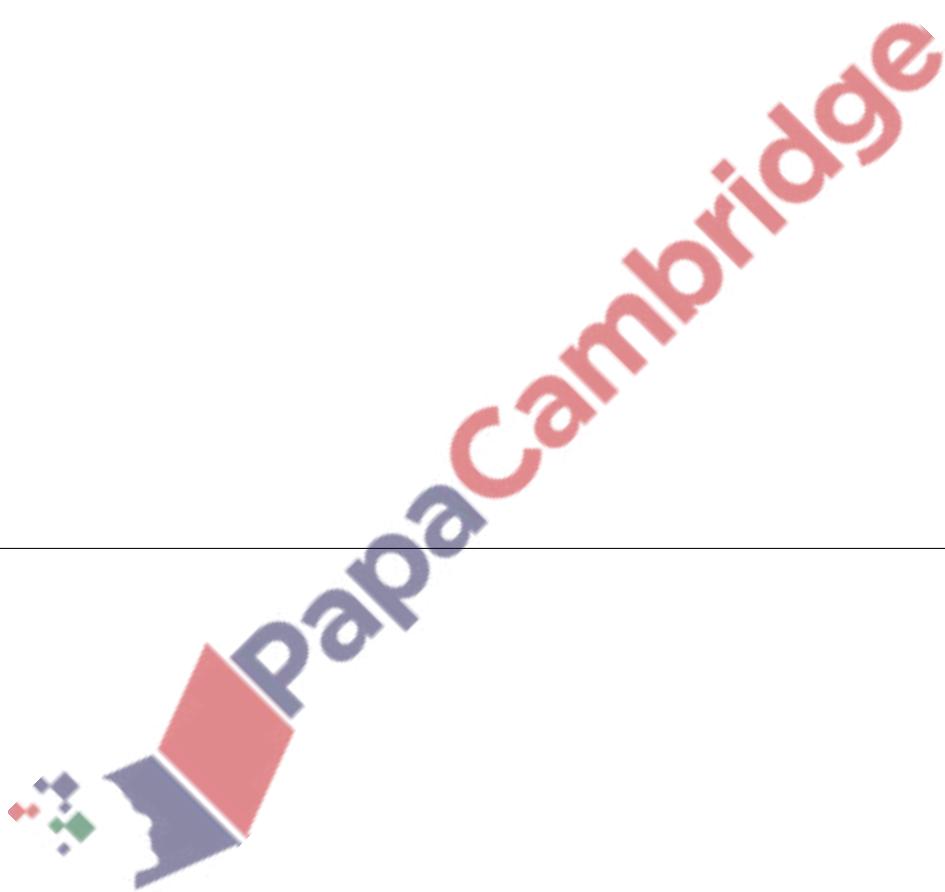
(a) find the least possible value of

$$3 \sin 2x \cot x + 5 \cos 2x + 8$$

as x varies, [4](b) find the exact value of $\int_{\frac{1}{8}\pi}^{\frac{1}{6}\pi} \operatorname{cosec} 4x \tan 2x \, dx$. [5]

176. 9709_s16_qp_21 Q: 7

(a) Find $\int \frac{1 + \cos^4 2x}{\cos^2 2x} dx$. [5]

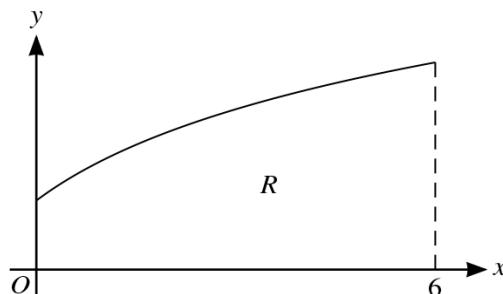
(b) Without using a calculator, find the exact value of $\int_4^{14} \left(2 + \frac{6}{3x-2}\right) dx$, giving your answer in the form $\ln(ae^b)$, where a and b are integers. [5]

177. 9709_s16_qp_22 Q: 6

(a) Find $\int \frac{4 + e^x}{2e^{2x}} dx$. [3]

(b) Without using a calculator, find $\int_2^{10} \frac{1}{2x+5} dx$, giving your answer in the form $\ln k$. [4]

(c)



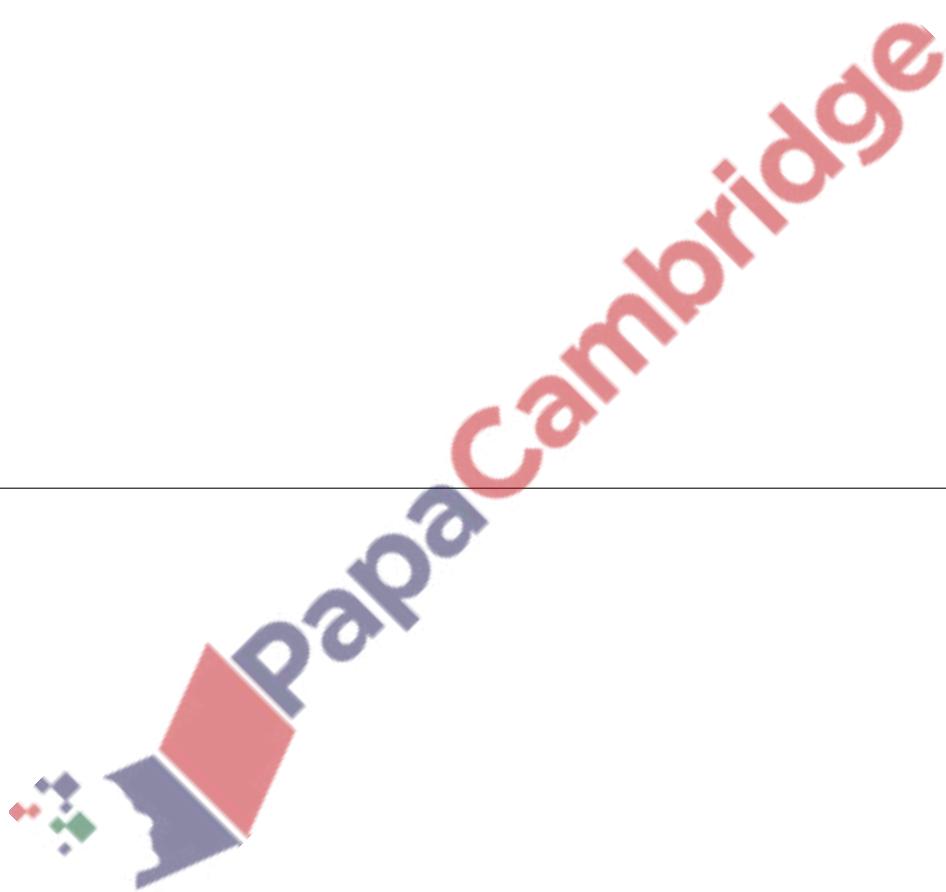
The diagram shows the curve $y = \log_{10}(x + 2)$ for $0 \leq x \leq 6$. The region bounded by the curve and the lines $x = 0$, $x = 6$ and $y = 0$ is denoted by R . Use the trapezium rule with 2 strips to find an estimate of the area of R , giving your answer correct to 1 decimal place. [3]



178. 9709_w16_qp_21 Q: 5

(i) Show that $\frac{\cos 2x + 9 \cos x + 5}{\cos x + 4} \equiv 2 \cos x + 1.$ [3]

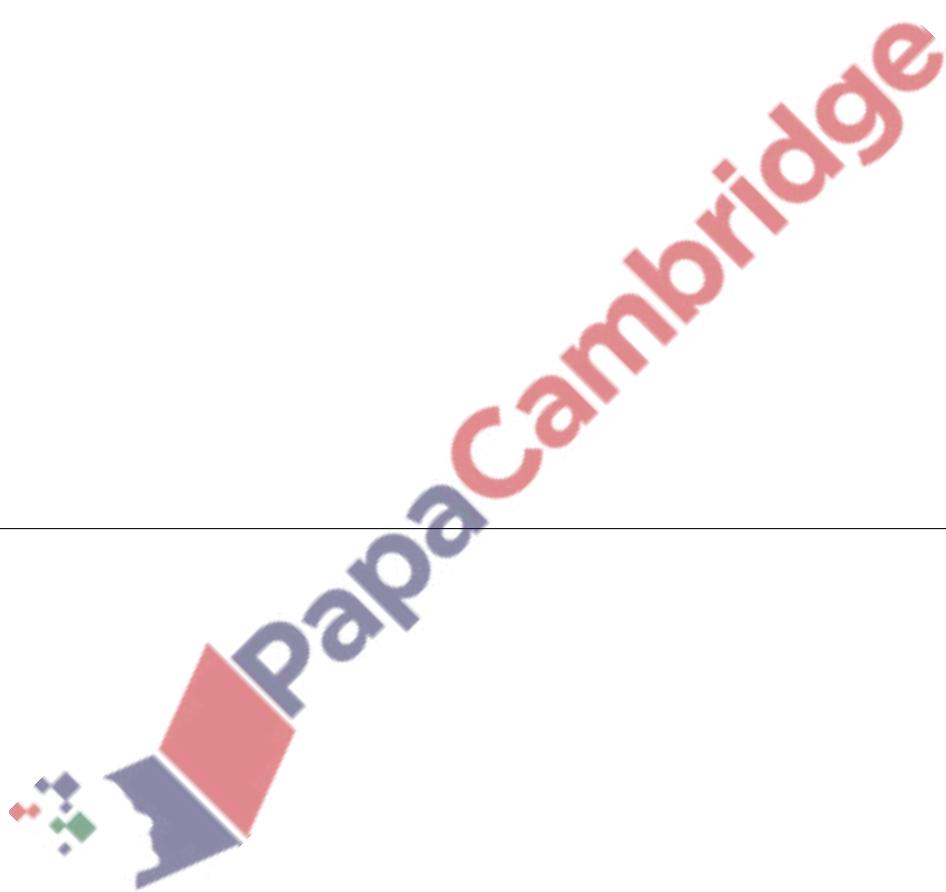
(ii) Hence find the exact value of $\int_{-\pi}^{\pi} \frac{\cos 4x + 9 \cos 2x + 5}{\cos 2x + 4} dx.$ [4]



179. 9709_w16_qp_22 Q: 3

The definite integral I is defined by $I = \int_0^2 (4e^{\frac{1}{2}x} + 3) dx$.

- (i) Show that $I = 8e - 2$. [3]
- (ii) Sketch the curve $y = 4e^{\frac{1}{2}x} + 3$ for $0 \leq x \leq 2$. [2]
- (iii) State whether an estimate of I obtained by using the trapezium rule will be more than or less than $8e - 2$. Justify your answer. [1]

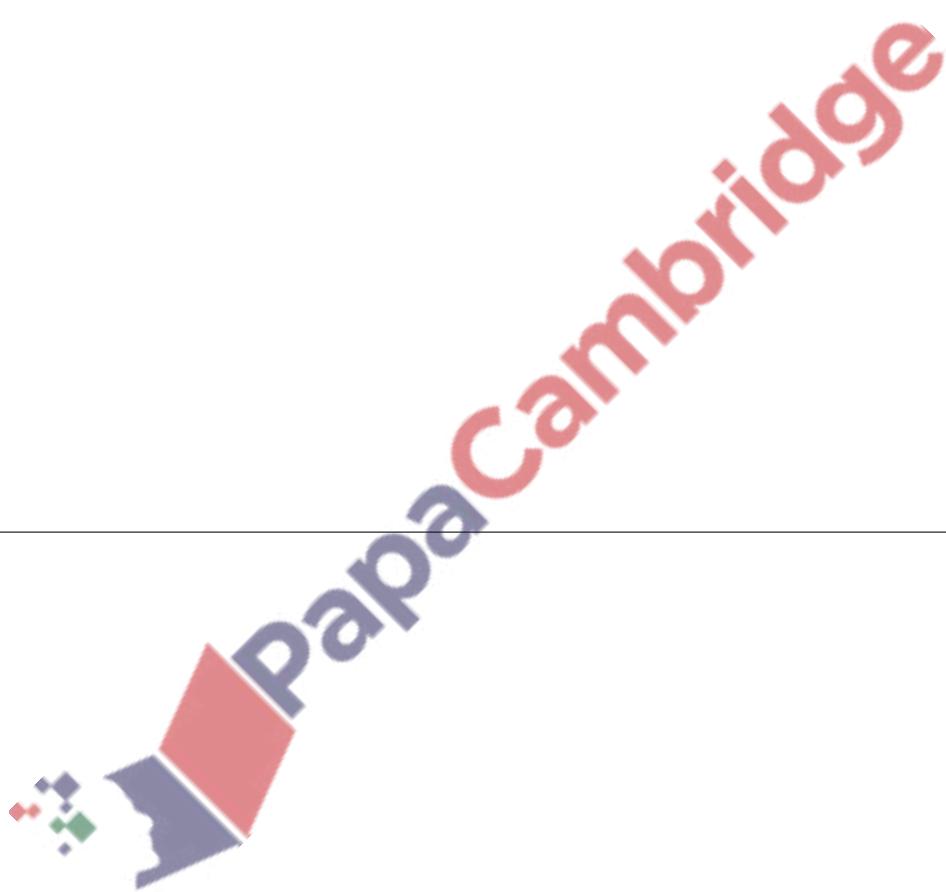


180. 9709_w16_qp_22 Q: 6

(i) Show that $\frac{\cos 2\theta}{1 + \cos 2\theta} \equiv 1 - \frac{1}{2} \sec^2 \theta.$ [2]

(ii) Solve the equation $\frac{\cos 2\alpha}{1 + \cos 2\alpha} = 13 + 5 \tan \alpha$ for $0 < \alpha < \pi.$ [4]

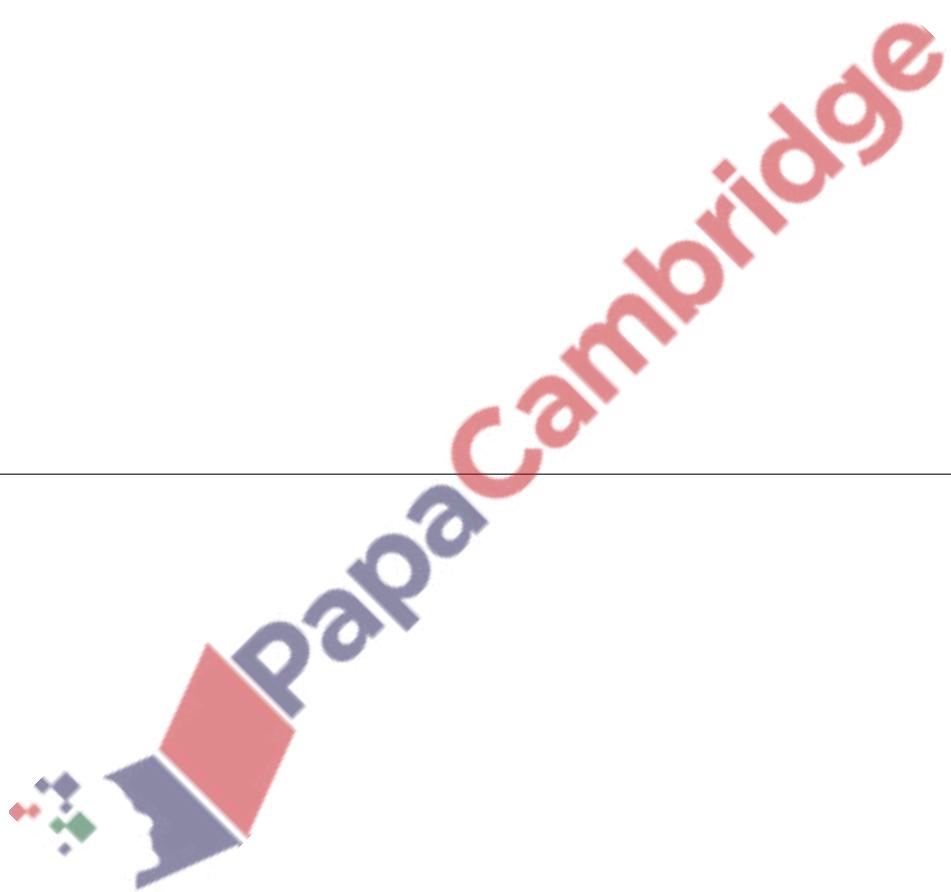
(iii) Find the exact value of $\int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi} \frac{\cos x}{1 + \cos x} dx.$ [4]



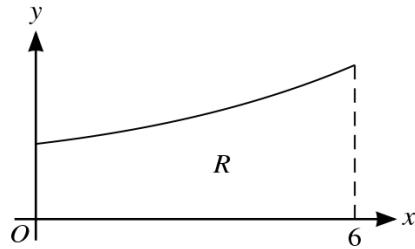
181. 9709_w16_qp_23 Q: 3

(i) Find $\int \tan^2 4x \, dx$. [2]

(ii) Without using a calculator, find the exact value of $\int_0^{\frac{1}{12}\pi} (4 \cos 2x + 6 \sin 3x) \, dx$. [3]



182. 9709_w16_qp_23 Q: 5



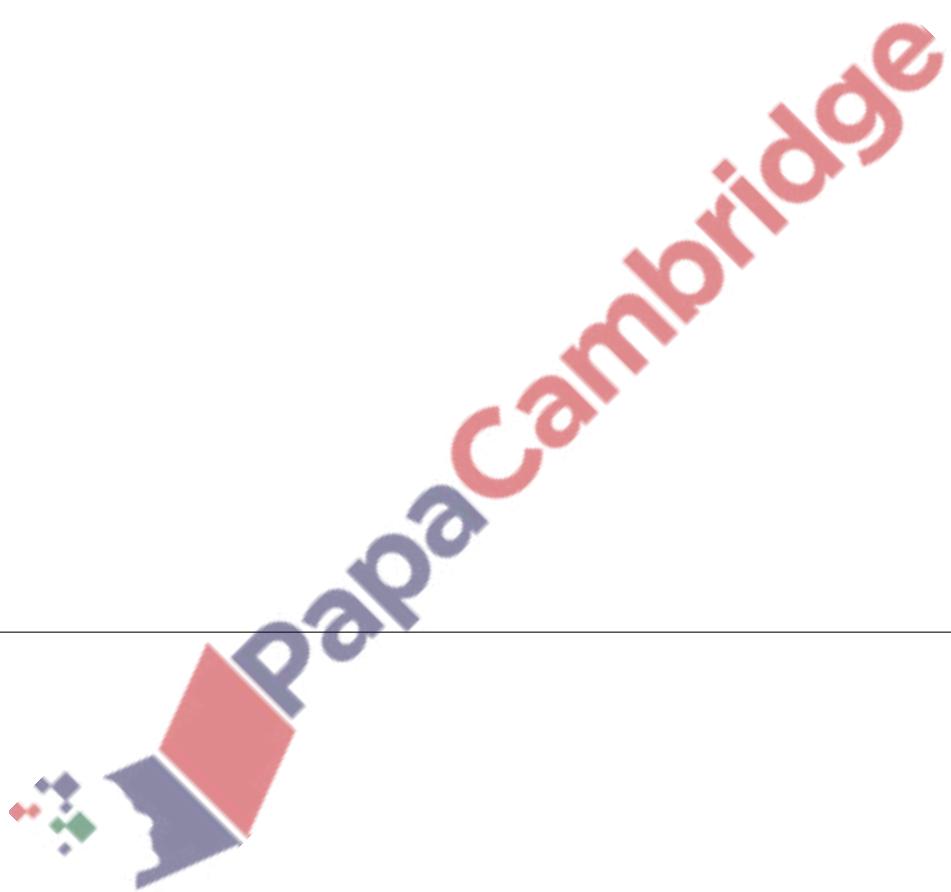
The diagram shows the curve $y = \sqrt{\left(1 + e^{\frac{1}{3}x}\right)}$ for $0 \leq x \leq 6$. The region bounded by the curve and the lines $x = 0$, $x = 6$ and $y = 0$ is denoted by R .

- (i) Use the trapezium rule with 2 strips to find an estimate of the area of R , giving your answer correct to 2 decimal places. [3]
- (ii) With reference to the diagram, explain why this estimate is greater than the exact area of R . [1]
- (iii) The region R is rotated completely about the x -axis. Find the exact volume of the solid produced. [4]

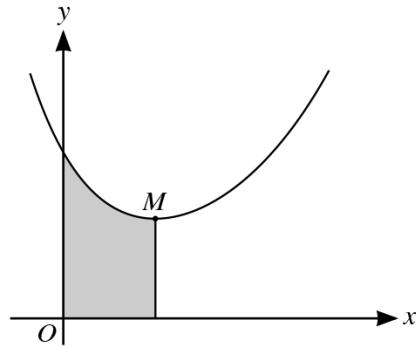


183. 9709_s15_qp_21 Q: 6

- (i) Prove that $2 \operatorname{cosec} 2\theta \tan \theta \equiv \sec^2 \theta$. [3]
- (ii) Hence
- (a) solve the equation $2 \operatorname{cosec} 2\theta \tan \theta = 5$ for $0 < \theta < \pi$, [3]
- (b) find the exact value of $\int_0^{\frac{1}{6}\pi} 2 \operatorname{cosec} 4x \tan 2x \, dx$. [4]

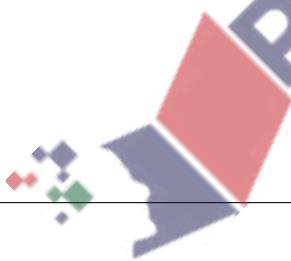


184. 9709_s15_qp_22 Q: 4

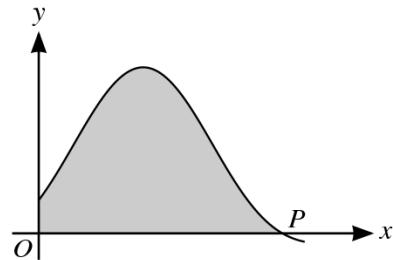


The diagram shows the curve $y = e^x + 4e^{-2x}$ and its minimum point M .

- (i) Show that the x -coordinate of M is $\ln 2$. [3]
- (ii) The region shaded in the diagram is enclosed by the curve and the lines $x = 0$, $x = \ln 2$ and $y = 0$. Use integration to show that the area of the shaded region is $\frac{5}{2}$. [4]



185. 9709_s15_qp_22 Q: 6



The diagram shows part of the curve with equation

$$y = 4 \sin^2 x + 8 \sin x + 3$$

and its point of intersection P with the x -axis.

- (i) Find the exact x -coordinate of P . [3]

- (ii) Show that the equation of the curve can be written

$$y = 5 + 8 \sin x - 2 \cos 2x,$$

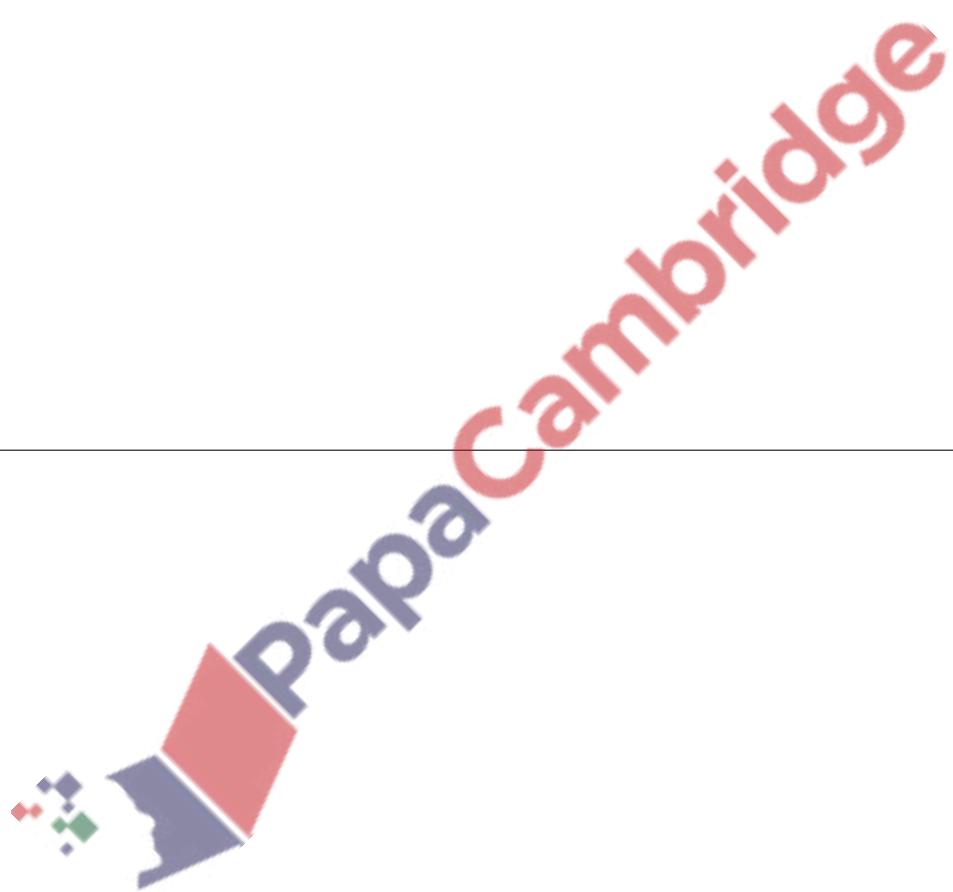
and use integration to find the exact area of the shaded region enclosed by the curve and the axes. [6]



186. 9709_w15_qp_21 Q: 5

(a) Find $\int (\tan^2 x + \sin 2x) dx$. [3]

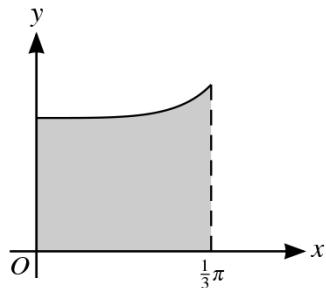
(b) Find the exact value of $\int_0^1 3e^{1-2x} dx$. [4]



187. 9709_w15_qp_22 Q: 7

- (i) Show that the exact value of $\int_0^{\frac{1}{3}\pi} \left(\cos^2 x + \frac{1}{\cos^2 x} \right) dx$ is $\frac{1}{6}\pi + \frac{9}{8}\sqrt{3}$. [6]

(ii)



The diagram shows the curve $y = \cos x + \frac{1}{\cos x}$ for $0 \leq x \leq \frac{1}{3}\pi$. The shaded region is bounded by the curve and the lines $x = 0$, $x = \frac{1}{3}\pi$ and $y = 0$. Find the exact volume of the solid obtained when the shaded region is rotated completely about the x -axis. [4]



188. 9709_w15_qp_23 Q: 1

Find the exact value of $\int_{-1}^{35} \frac{3}{2x+5} dx$, giving the answer in the form $\ln k$. [5]

